Basic Number Theory

Summary of Important Facts for chapter 8:

1. Types of Numbers
   a. Integers ($\mathbb{Z}$)
      ... , -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

   b. Natural Numbers ($\mathbb{N}$)
      1, 2, 3, 4, 5, ...

   c. Rational Numbers ($\mathbb{Q}$)
      i. All numbers that can be written as a fraction $^a/_b$ where a and b are integers and $b \neq 0$.
      ii. $(^a/_b) \div (^c/_d) = (^a/_b) \times (^d/_c)$

   d. Irrational Numbers
      i. Numbers which don't have terminating decimals or repeating decimals. For example, the numbers π and e are irrational.
      ii. Simplifying a radical
      iii. Rationalizing a radical

   e. Real Numbers ($\mathbb{R}$)
      These include all of the above. The only type of numbers which are not real are imaginary (ie: imaginary numbers have "i" in them)

2. Sequences
   a. Arithmetic
      i. The difference between each term is some constant number
      ii. d = difference between each term
      iii. $a_{n+1} = d + a_n$

   b. Geometric
      i. The difference between each term is not a constant number
      ii. Each term is some constant multiple of the previous term.
      iii. r = the common ratio between each term
      iv. $a_{n+1} = r \cdot a_n$

   c. Fibonacci
      i. 1, 1, 2, 3, 5, 8, 13, ...
      ii. $F_{n+1} = F_n + F_{n-1}$ (each term is the sum of the previous two)

   d. Other
      There is some other pattern.
      For example: 1, 4, 9, 16, 25, 36, 49, ... , n^2

The n^{th} term in the Sequence

The n^{th} term in a sequence is denoted by a_n. Notice that the subscript is "n" since it is the n^{th} term. There are two ways to write what a_n is. One way writes a_n in terms of "n". This means that you don't have to know what any of the previous terms are and can find a_n directly. The other way writes a_n in terms of the previous terms of the sequence. The advantage of this way is that you can easily see what the pattern of the sequence is.

1. **Recursive Form:** a_n is written in terms of the previous terms of the sequence

   Assumptions: to find a_n we must have the previous terms

   Examples:

   1. 1, 3, 5, 7, 9, ...

      $a_n = a_{n-1} + 2$ where a_{n-1} is the term before a_n

   2. 2, 4, 8, 16, 32, ...

      $a_n = 2 \cdot a_{n-1}$

   3. -1, -1, 0, 2, 5, 9, ...

      $a_n = a_{n-1} + (n-2)$ notice that you're adding 0, 1, 2, 3, etc to get the next term

2. **Closed Form:** a_n is written in terms of "n"

   Assumptions: to find a_n we look to find a pattern which corresponds to the term number. In all examples, we assume that the first term is a_1.

   Strategies:

   - look at what must be added to get the next term
   - look at the factors of each term
   - try multiplying all terms by some number
   - try adding some number to all terms

   Examples:

   4. 1, 3, 5, 7, 9, ... $\Rightarrow$ 2, 4, 6, 8, 10, ... $\Rightarrow$ $2\cdot1, 2\cdot2, 2\cdot3, 2\cdot4, 2\cdot5, \ldots$

      $a_n = 2n - 1$

   5. 2, 4, 8, 16, 32, ... $\Rightarrow$ $2^1, 2^2, 2^3, 2^4, 2^5, \ldots$

      $a_n = 2^n$

   6. -1, -1, 0, 2, 5, 9, ... $\Rightarrow$ -2, -2, 0, 4, 10, 18, ... $\Rightarrow$ $-2\cdot1, -1\cdot2, 0\cdot3, 1\cdot4, 2\cdot5, 3\cdot6, \ldots$

      $a_n = n(n-3)/2$