Key

QUIZ 12

For questions 1 - 3, identity a convergence test (which will not fail) to use to determine whether the series will converge or diverge. Then use the test to determine the convergence of the series.

1. (3 points)

$$\sum_{n=1}^{\infty} \frac{(-1)^n(n+4)}{1-2n}$$

$$\lim_{n\to\infty} \left|\frac{(-1)^n(n+4)}{1-2n}\right| = \left|\lim_{n\to\infty} \frac{n+4}{1-2n}\right|$$

$$= \left|\lim_{n\to\infty} \frac{n+4}{-2n+1}\right| = \left|-\tfrac{1}{2}\right| = \tfrac{1}{2} \neq 0$$

Alternating Series

Diverges

2. (3 points)

$$\sum_{n=1}^{\infty} \frac{(-3)^n}{n^2}$$

$$L = \lim_{n\to\infty} \left|\frac{\frac{(-3)^{n+1}}{(n+1)^2}}{\frac{(-3)^n}{n^2}}\right| = \lim_{n\to\infty} \left|\frac{(-3)^{n+1}}{(n+1)^2} \cdot \frac{n^2}{(-3)^n}\right|$$

$$= \lim_{n\to\infty} \left|\frac{-3n^2}{n^2+2n+1}\right| = 3$$

Alternating ~~Series~~ Series OR Ratio Test

So, $L = 3 > 1$

Diverges

3. (3 points)

$$\sum_{n=1}^{\infty} \left(\frac{n+4}{1-2n}\right)^n$$

$$\lim_{n\to\infty} \left[\left(\frac{n+4}{1-2n}\right)^n\right]^{1/n} = \lim_{n\to\infty} \left(\frac{n+4}{1-2n}\right)$$

$$= \left|\lim_{n\to\infty} \frac{n+4}{1-2n}\right| = \left|-\tfrac{1}{2}\right| = \tfrac{1}{2} < 1$$

Root Test

Converges

4. (1 point) Power series are not polynomials. However, polynomials can be considered a special type of power series. Explain why this second statement is true. (Note: if you give an example, be sure to explain what's going on.)

Let $p(x) = 1 + 2x - 3x^2$. This is a polynomial.

Rewritten as a power series:

$1 + 2x - 3x^2 + 0x^3 + 0x^4 + 0x^5 + \dots$ where $C_n = 0$ for all $n \geq 3$