Key

QUIZ 13

Find the radius of convergence (R) and the interval of convergence (IC) of the given power series. (Hint: don't forget to check the endpoints of the interval, and be sure to state any convergence test that you use.)

$$\sum_{n=0}^{\infty} \frac{(-1)^{n-1}(x+2)^n}{7^n(n-3)}$$

a_{n+1} $\qquad$ $\frac{1}{a_n}$

(3)

$$\lim_{n\to\infty} \left| \frac{(-1)^n(x+2)^{n+1}}{7^{n+1}(n-2)} \cdot \frac{7^n(n-3)}{(-1)^{n-1}(x+2)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)(x+2)(n-3)}{7(n-2)} \right|$$

$$= |x+2| \cdot \lim_{n\to\infty} \frac{n-3}{7n-14} = |x+2| \cdot \frac{1}{7}$$

(1) $L = |x+2| \cdot \frac{1}{7}$

to converge by Ratio Test: $L < 1$

$$|x+2| \cdot \frac{1}{7} < 1 \implies |x+2| < 7 \implies -9 < x < 5$$

(2) at $x=-9$: $\sum \frac{(-1)^{n-1}(-9+2)^n}{7^n(n-3)} = \sum \frac{(-1)^{n-1}(-7)^n}{7^n(n-3)} = \sum \frac{(-1)^{2n-1}}{n-3} = \sum \frac{-1}{n-3}$

use LCT: compare with $\sum \frac{-1}{n}$ p=1 so this new one will diverge

$$\lim_{n\to\infty} \frac{\left(\frac{-1}{n-3}\right)}{\left(-1/n\right)} = \lim_{n\to\infty} \frac{n}{n-3} = 1$$

$\therefore$ diverges at $x=-9$

(2) at $x=5$ $\sum \frac{(-1)^{n-1}(5+2)^n}{7^n(n-3)} = \sum \frac{(-1)^{n-1}}{n-3}$

use Alternating series test; $\lim_{n\to\infty} \frac{1}{n-3} = 0$ Converges

(1) The radius of converge: $R = 7$

(1) The interval of converge: $-9 < x \leq 5$

or $(-9, 5]$