

QUIZ 16

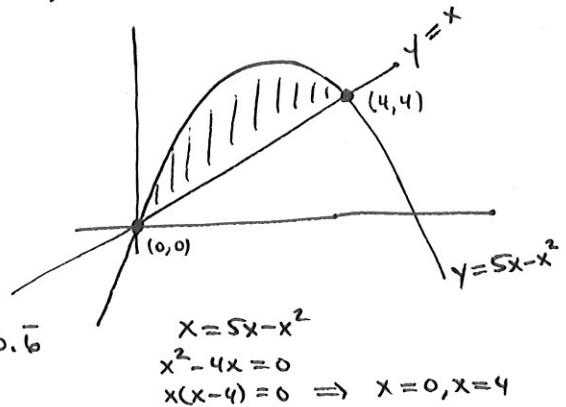
1. (4 points) Find the area of the region bounded by $y = 5x - x^2$ and $y = x$.

$$\text{Area} = \int_a^b (\text{top}) - (\text{bottom}) \, dx$$

$$= \int_0^4 (5x - x^2) - x \, dx$$

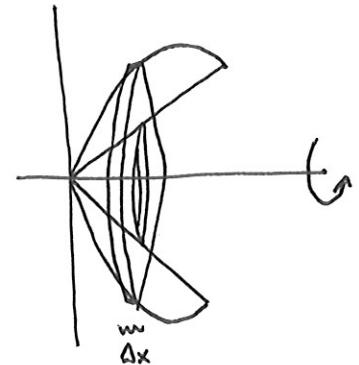
$$= \int_0^4 4x - x^2 \, dx$$

$$= 2x^2 - \frac{1}{3}x^3 \Big|_0^4 = 8 - \frac{64}{3} = \frac{32}{3} = 10.\overline{6}$$



2. (6 points) Find the volume of the region described in #1 when it is rotated about the x -axis.

$$\begin{aligned} V &= \int_a^b A(x) \, dx \\ &= \int_0^4 \pi \left[(5x - x^2)^2 - x^2 \right] \, dx \\ &= \pi \int_0^4 25x^2 - 10x^3 + x^4 - x^2 \, dx \\ &= \pi \left[8x^3 - \frac{5}{2}x^4 + \frac{1}{5}x^5 \right]_0^4 \\ &= \frac{384\pi}{5} \\ &\approx 241.2743158 \end{aligned}$$

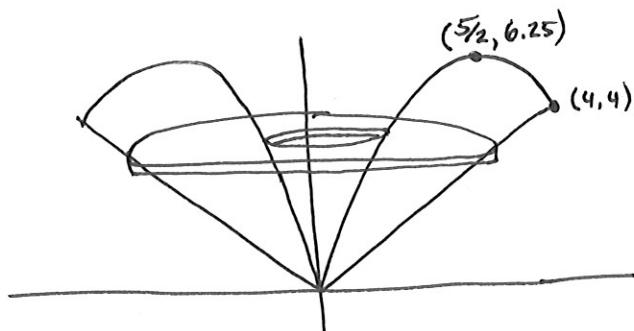


$$R = 5x - x^2$$

$$r = x$$

$$\begin{aligned} A(x) &= \pi R^2 - \pi r^2 \\ &= \pi [R^2 - r^2] \end{aligned}$$

3. (extra credit) Find the volume of the region described in #1 when it is rotated about the y -axis.



can't use slices here
b/c R & r would
come from the same
function