QUIZ 17
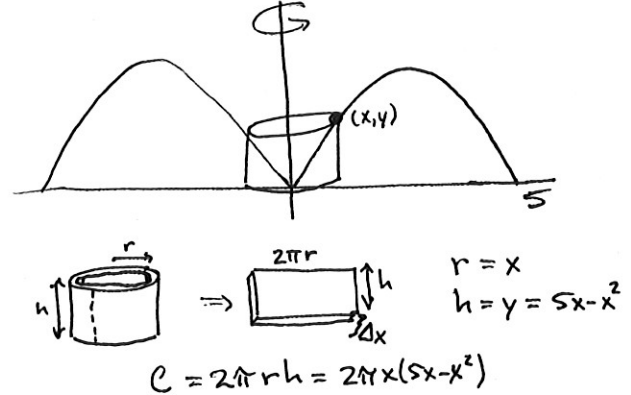
1. (4 points)

   a. Find the volume of the region which is bounded by $y = 5x - x^2$ and $y = 0$ when it is rotated about the y-axis.

   (4)

   $$V = \int_0^5 2\pi x(5x - x^2)\, dx$$

   $$= 2\pi \int_0^5 5x^2 - x^3\, dx$$

   $$= 2\pi \left[\frac{5}{3}x^3 - \frac{1}{4}x^4 \right] \Big|_0^5$$

   $$= 2\pi \left(\frac{625}{3} - \frac{625}{4} \right)$$

   $$= \frac{625\pi}{6} \approx 327.2492347$$

   $r = x$
   $h = y = 5x - x^2$

   $C = 2\pi r h = 2\pi x(5x - x^2)$

   b. Explain why using the slicing (or cross-section) method would fail in this situation.

   (1)

   the slice would be a washer but we only have the one function

2. (5 points) Find the arc length of the curve $y = 2x - 5$ on $-1 \le x \le 3$.

   a. Use the arc length formula/integral to compute this value.

   (3)

   $$L = \int_a^b \sqrt{(x')^2 + (y')^2} = \int_{-1}^3 \sqrt{1 + (2)^2}\, dx$$

   $$= \int_{-1}^3 \sqrt{5}\, dx = \sqrt{5}\, x \Big|_{-1}^3 = 3\sqrt{5} - (-\sqrt{5}) = 4\sqrt{5}$$

   $$\approx 8.94427191$$

   b. Use the distance formula to compute this value.

   (1)

   $x_1 = -1$
   $y_1 = 2(-1) - 5 = -7$

   $x_2 = 3$
   $y_2 = 2(3) - 5 = 1$

   $$d = \sqrt{(-1 - 3)^2 + (-7 - 1)^2}$$

   $$= \sqrt{16 + 64} = \sqrt{80} = \sqrt{16 \cdot 5}$$

   $$= 4\sqrt{5}$$

   c. Why should the two answers from part (a) and (b) be the same?

   (1)

   b/c $y = 2x - 5$ is a straight line