Key

READING ASSESSMENT

(sections 8.4 – 8.5)

Topics Covered: The Root Test and an introduction to Power Series.

Directions: Read about the topics above. If you have Stewart's textbook (1st or 2nd edition), this will be the end of section 8.4 as well as section 8.5. Then answer the questions below. Note: there are fill in the blank questions, definition questions, and an actual question or two to work.

The **Root Test** is one of the most powerful of the series tests for convergence (the other is the integral test). However, often it is not the most used test simply because of how L is calculated. For the root test, what is L? (Complete the formula below.)

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} (|a_n|)^{1/n}$$

Once L has been calculated, just like with the Ratio Test, we determine whether the series converges or not based off of how L compares to the number 1. Indicate below when the series will converge, diverge, or that the test fails:

$L < 1$ Converges

$L = 1$ test fails

$L > 1$ Diverges

Determine whether the following series converges or diverges using the Root Test:

$$\sum_{n=1}^{\infty} \left(\frac{2n}{n+1}\right)^n \qquad L = \lim_{n\to\infty} \left(\frac{2n}{n+1}\right) = 2 > 1$$

Diverges

When a **Power Series** is written out (ie: not using sigma notation), it looks very similar to a polynomial except that a power series is infinite whereas a polynomial has a degree, which means that it must terminate eventually. You can think of a power series as a function where when you plug in values of x, you get back different series. Just like with regular series, we still want to know whether a power series will converge or diverge. EXCEPT that for a power series, we want to fill all of the values that can be plugged in to the power series for x which result in a convergent series.

To determine which values of x will cause a power series to converge, we could use many of the previous tests that we have learned about; however, it is standard to use the Ratio Test. Consider the power series below. In this case, we could use the Geometric Test or the Ratio Test.

$$\sum_{n=0}^{\infty} 2x^n \qquad L = \lim_{n\to\infty} \left|\frac{2x^{n+1}}{2x^n}\right| = \lim_{n\to\infty} |x| = |x| < 1$$

If we apply the Ratio Test, then $L =$ $|x|$ (don't forget the absolute values). From the Ratio test, we know that if $L < 1$, then the series converges. So we get that -1 $< x <$ 1. But the Ratio test fails when $L = 1$. This means the last step is to check what happens at the endpoints of the interval you listed above (we'll talk more about this in class).

endpoints: at $x = -1$, $\sum 2(-1)^n$ alternating, $\lim_{n\to\infty} |2(-1)^n| = \lim_{n\to\infty} 2 = 2 \neq 0$, Div.

at $x = 1$, $\sum 2(1)^n = \sum 2$ Test for Divergence. Diverges