Key

READING ASSESSMENT

(sections 8.5 – 8.6)

Topics Covered: Finding the Interval of Convergence of a Power Series and building new Power series.

Directions: Read about the topics above. If you have Stewart's textbook (1st or 2nd edition), this will be section 8.5 as well as section 8.6. Then answer the questions below.

When working with a **Power Series**, the most fundamental things that we care about are the interval of convergence and the radius of convergence. These two are closely related. Recall that the standard format of a power series is

$$\sum_{n=0}^{\infty} c_n(x-a)^n$$

If the radius of convergence is R, then the interval of convergence is $|x-a|<R$ which is equivalent to $\underline{a-R} < x < \underline{a+R}$. There are actually 4 possibilities since the inequalities could be < or ≤ hence we get options: $(a-R, a+R), (a-R, a+R], [a-R, a+R),$ *or* $[a-R, a+R]$. Warning: this is true for $0<R<\infty$. If $R=0$ or $R=\infty$, the interval of convergence looks quite different.

To determine which endpoint(s) need to be included in the interval of convergence, plug each endpoint in for x in the power series. You will now have a normal series. Then use one of the convergence tests to determine whether the resulting series converges or diverges. If the series <u>converges</u>, you will want to include that endpoint in the interval of convergence.

$$\lim_{n\to\infty}\left|\frac{2^{n+1}(x-1)^{n+1}}{(n+1)^2}\cdot\frac{n^2}{2^n(x-1)^n}\right| = \lim_{n\to\infty}\left|\frac{2(x-1)n^2}{n^2+2n+1}\right| = 2|x-1|$$

Example: consider the following power series $\sum \frac{2^n(x-1)^n}{n^2}$

so $2|x-1|<1 \Rightarrow |x-1|<1/2 \Rightarrow -\frac{1}{2}<x-1<1/2$

$1/2 < x < 3/2$

If we apply the Ratio Test, then $L = \underline{2|x-1|}$ (don't forget the absolute values). From the Ratio test, we know that if $L<1$, the series converges. So we get that $|x-1| < \underline{1/2}$, which is the same as $\frac{1}{2} < x < \underline{3/2}$. But the Ratio test fails when $L=1$. So we need to check what happens at the endpoints of the interval. The left endpoint is $x=\frac{1}{2}$. If we plug this number into the power series, we get $\sum\frac{(-1)^n}{n^2}$ This series <u>Converges</u> by the alternating series test. The right endpoint is $x = \underline{3/2}$. If we plug this number into the power series, we get $\sum \frac{1}{n^2}$ This series converges by the <u>p-series test</u> $(p=2)$ test. Putting everything above together, the interval of convergence is: $1/2 \le x \le 3/2$ or $[1/2, 3/2]$

The power series: $\sum_{n=0}^{\infty} x^n = \frac{1}{1-x}$ when $-1<x<1$. (We know this by the geometric series test.) If you know that a function equals a certain power series, you can use that information to **find the power series of a related function**. For example, the power series of $\frac{1}{1-2x}$ is $\sum_{n=0}^{\infty} 2^n x^n$. Why? Compare the functions: $\frac{1}{1-x}$ and $\frac{1}{1-2x}$. In place of the x in the first function, the second function has $2x$. So to get the power series of $\frac{1}{1-2x}$, we plug $2x$ in for x in the power series of $\underline{\frac{1}{1-x} = \sum x^n}$. Note, $(2x)^n = \underline{2^n x^n}$

Using this same technique, the power series of $\frac{1}{1-x^2}$ will be: $\sum_{n=0}^{\infty}(x^2)^n = \sum_{n=0}^{\infty} x^{2n}$