Homework Set 1

Review Topics

Simplify the following expressions:

1. $\left(\frac{xx^{-3}y^5z^2}{x^2y^3zz^{-2}}\right)^2$ $= \left(\frac{x^2z^3}{x^4}\right)^2 = \frac{y^4z^6}{x^8}$

2. $\frac{2}{1+\frac{x}{x-3}}$ $= \frac{2}{\left[\frac{(x-3)+x}{x-3}\right]} = \frac{2(x-3)}{2x-3} = \frac{2x-6}{2x-3}$

3. $\frac{4x^2+x-6}{x^2+3x+2} - \frac{3x}{x+1} + \frac{5}{x+2}$ $= \frac{4x^2+x-6}{(x+1)(x+2)} - \frac{3x(x+2)}{(x+1)(x+2)} + \frac{5(x+1)}{(x+1)(x+2)}$

$= \frac{4x^2+x-6-(3x^2+6x)+5x+5}{(x+1)(x+2)} = \frac{x^2-1}{(x+1)(x+2)} = \frac{(x+1)(x-1)}{(x+1)(x+2)} = \frac{x-1}{x+2}$

4. Use rules for logarithms to expand: $\ln(\frac{(x-1)^2}{x\sqrt{x}})$

$= \ln(x-1)^2 - \ln x^{3/2}$

$= 2\ln(x-1) - \frac{3}{2}\ln x$

5. Use an appropriate method to compute: $\lim_{x\to\infty}(1+3x)^{1/x}$ $= e^3$

$\lim_{x\to\infty} \ln(1+3x)^{1/x} = \lim_{x\to\infty} \frac{1}{x}\ln(1+3x) = \lim_{x\to\infty} \frac{\ln(1+3x)}{x} = \lim_{x\to\infty} \frac{\left[\frac{3}{1+3x}\right]}{1} = 3$

Use Trig identities to simplify the following epressions:

6. $\frac{\sin(x)}{\csc(x)} + \frac{\cos(x)}{\sec(x)}$

$= \sin^2 x + \cos^2 x = 1$

7. $3\tan^2(\theta) - 3\sec^2(\theta)$

$= \frac{3\sin^2\theta}{\cos^2\theta} - \frac{3}{\cos^2\theta} = \frac{3(\sin^2\theta - 1)}{\cos^2\theta} = \frac{3(-\cos^2\theta)}{\cos^2\theta} = 3$

8. Rewrite the following angles in degrees in radians:
   (a) $30° = \pi/6$
   (b) $45° = \pi/4$
   (c) $60° = \pi/3$
   (d) $90° = \pi/2$
   (e) $135° = 3\pi/4$

Compute the derivative for each of the given functions:

9. $y = 2x^{35} - 13x^{11} + 2x^3 - 5$

$$y' = 70x^{34} - 143x^{10} + 6x^2$$

10. $f(x) = 4e^{1-x^2}$

$$f'(x) = 4(-2x)e^{1-x^2} = -8xe^{1-x^2}$$

11. $g(t) = \arctan(5t)$

$$g'(t) = \frac{5}{1+(5t)^2} = \frac{5}{1+25t^2}$$

12. $y = \ln(x^7 - 3x^4 + 6x)$

$$\frac{dy}{dx} = \frac{7x^6 - 12x^3 + 6}{x^7 - 3x^4 + 6x}$$

13. $h(u) = u \cdot \sin(u)$

$$h'(u) = \sin u + u\cos u$$

14. $y = \frac{t^5-1}{t^5+1}$

$$\frac{dy}{dt} = \frac{5t^4(t^5+1) - (t^5-1)\cdot 5t^4}{(t^5+1)^2} = \frac{10t^4}{(t^5+1)^2}$$

15. $k(x) = \cos(2x)\left(\frac{2x-1}{x+3}\right)^7$

$$K'(x) = -2\sin 2x\left(\frac{2x-1}{x+3}\right)^7 + 7\cos(2x)\left(\frac{2x-1}{x+3}\right)^6 \cdot \frac{2(x+3)-(2x-1)}{(x+3)^2}$$

$$= -2\sin 2x\left(\frac{2x-1}{x+3}\right)^7 + 7\cos 2x\left(\frac{2x-1}{x+3}\right)^6 \cdot \frac{7}{(x+3)^2}$$

or $$= -2\sin 2x\left(\frac{2x-1}{x+3}\right)^7 + \frac{49\cos 2x\,(2x-1)^6}{(x+3)^8}$$