due: April 9

Homework Set 3

Appendix C: Properties of Summations

Write each sum in its expanded form:

1. $\sum_{k=0}^{3} 2^{3k} = 2^0 + 2^3 + 2^6 + 2^9 = 1 + 8 + 64 + 512 = 585$

2. $\sum_{i=4}^{7} x^i = x^4 + x^5 + x^6 + x^7$

Write each sum in sigma notation:

3. $2 + 4 + 6 + 8 + \ldots + 2n = \sum_{k=1}^{n} 2k$

4. $\frac{5}{8} + \frac{6}{9} + \frac{7}{10} + \frac{8}{11} + \ldots + \frac{21}{24} = \sum_{k=5}^{21} \frac{k}{k+3}$

Find the value of each sum. (Note: some of your answers may have n in them.)

5. $\sum_{i=1}^{50} 3 = 3 + 3 + \ldots + 3 = 3(50) = 150$

6. $\sum_{j=0}^{2} (2^j + j^2) = (2^0 + 0^2) + (2^1 + 1^2) + (2^2 + 2^2) = 1 + 3 + 8 = 12$

7. $\sum_{k=1}^{n} (7 - 2k) = \sum_{k=1}^{n} 7 - 2\sum_{k=1}^{n} k = 7n - 2 \cdot \frac{n(n+1)}{2} = 7n - (n^2 + n) = 6n - n^2$

8. $\sum_{k=1}^{n} (k-1)(k+2) = \sum_{k=1}^{n} (k^2 + k - 2) = \sum_{k=1}^{n} k^2 + \sum_{k=1}^{n} k - \sum_{k=1}^{n} 2 = \frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} - 2n$

$$= \frac{n(n+1)(2n+1) + 3n(n+1) - 12n}{6} = \frac{2n^3 + 6n^2 - 8n}{6} = \frac{n(n+4)(n-1)}{3}$$

Calculate the following limit.

9. $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n}\left(\frac{i}{n}\right)^2 = \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2 = \lim_{n\to\infty} \frac{1}{n^3} \cdot \frac{n(n+1)(2n+1)}{6} = \lim_{n\to\infty} \frac{2n^3 + \ldots}{6n^3} = 1/3$

or this is $\int_0^1 x^2\,dx = \frac{1}{3}x^3\Big|_0^1 = 1/3$

10. $\lim_{n\to\infty} \sum_{k=1}^{n} \frac{3}{n}\left[2 - \frac{k}{n}\right] =$

$$= \lim_{n\to\infty} \left[\frac{6}{n}\sum_{k=1}^{n} 1 - \frac{3}{n^2}\sum_{k=1}^{n} k\right] = \lim_{n\to\infty} \left[\frac{6}{n} \cdot n - \frac{3}{n^2} \cdot \frac{n(n+1)}{2}\right]$$

$$= \lim_{n\to\infty} \left[6 - \frac{3n^2 + 3n}{2n^2}\right] = 6 - 3/2 = 9/2$$

or this is $3\int_0^1 (2 - x)\,dx = 3\left[2x - \frac{1}{2}x^2\right]_0^1 = 3(2 - 1/2) = 9/2$