due: April 16

Homework Set 4

Sect 8.4: the Root Test

Use the Root Test to determine whether each series is convergent or divergent. If the test fails, state this, and then determine which test should have been used to determine convergence.

1. $\sum_{n=0}^{\infty} \left(\frac{2n-1}{n+7}\right)^{n/2}$

$$L = \lim_{n\to\infty} \left(\frac{2n-1}{n+1}\right)^{1/2} = \lim_{n\to\infty} \sqrt{2} = \sqrt{2} > 1$$

Diverges

2. $\sum_{n=0}^{\infty} \left(\frac{n+1}{3n-1}\right)^{n}$

$$L = \lim_{n\to\infty} \frac{n+1}{3n-1} = 1/3 < 1$$

Converges

3. $\sum_{n=0}^{\infty} \left(\frac{2n-5}{2-3n}\right)^{3n}$

$$L = \lim_{n\to\infty} \left|\frac{2n-5}{2-3n}\right|^3 = \left|\left(-\frac{2}{3}\right)^3\right| = 8/27 < 1$$

Converges

4. $\sum_{n=1}^{\infty} \frac{4^{3n}}{n^n}$

$$L = \lim_{n\to\infty} \frac{4^3}{n} = 0 < 1$$

Converges

5. $\sum_{n=1}^{\infty} \frac{5^{n/2}(-n)^{4n}}{(1+e^n)^n}$

$$L = \lim_{n\to\infty} \frac{5^{1/2} n^4}{1+e^n} = \lim_{n\to\infty} \frac{4\sqrt{5} n^3}{e^n} = \lim_{n\to\infty} \frac{12\sqrt{5} n^2}{e^n}$$

$$= \lim_{n\to\infty} \frac{24\sqrt{5}\, n}{e^n} = \lim_{n\to\infty} \frac{24\sqrt{5}}{e^n} = 0 < 1$$

Converges

6. $\sum_{n=0}^{\infty} (\sqrt[n]{n}+1)^n$

$$L = \lim_{n\to\infty} (\sqrt[n]{n}+1) = 1 + \lim_{n\to\infty} n^{1/n} = 1 + e^0 = 2 > 1$$

$$\lim_{n\to\infty} \ln(n^{1/n}) = \lim_{n\to\infty} \tfrac{1}{n}\ln(n) = \lim_{n\to\infty} \frac{\ln(n)}{n} = \lim_{n\to\infty} \frac{1/n}{1} = 0$$

Diverges