Homework Set 5

Sect 8.8: Applications of Taylor Polynomials

1. Consider $f(x) = 1/x$ on $0.8 \leq x \leq 1.2$.
   (a) Appoximate f with a Taylor polynomial with degree $n = 3$ at the number $a = 1$

   (b) Use Taylor's Formula to estimate the accuracy of the approximation $T_3(x) \approx f(x)$ when x is in the given interval.

2. Consider $f(x) = \cos(x)$ on $0 \leq x \leq 2\pi/3$.
   (a) Appoximate f with a Taylor polynomial with degree $n = 4$ at the number $a = \pi/3$

   (b) Use Taylor's Formula to estimate the accuracy of the approximation $T_4(x) \approx f(x)$ when x is in the given interval.

   (c) Use the information from part (b) to estmiate $\cos 78^\circ$ correct to 5 decimal places.

3. Consider the function e^x.

   (a) Use Taylor's Formula to determine the number of terms of the Maclaurin series for e^x that should be used to estimate $e^{0.1}$ to within 0.00001.

   (b) Using the information in part (a), estimate $e^{0.1}$ to within 0.00001.

4. Suppose that you know that $f^{(n)}(4) = \frac{(-1)^n n!}{3^n(n+1)}$ and the Taylor series of f centered at 4 converges to $f(x)$ for all x in the interval of convergence. Show that the fifth-degree Taylor polynomial approximates $f(5)$ with error less than 0.0002.