Full solution is provided

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WeBWorK assignment number Series_Practice is due : 11/30/2010 at 11:30pm EST.

The primary purpose of this set is to practice with the convergence or divergence of a series. It has 6 problems that contains 6 series each. You have **ONLY 2 ATTEMPTS** to get a problem correct and each problem is worth 3 points.

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1. (3 pts) UNCC1242/EssentialCalculus-Stewart-Sec8.4.Extra1.pg

NOTE: Only 2 attempts are allowed for the whole problem.

Match each of the following series with the correct statement:

A. The series is absolutely convergent.
C. The series is conditionally convergent.
D. The series diverges.

___1. $\sum_{n=1}^{\infty}(-1)^{n+1}\frac{4^n}{n!}$

___2. $\sum_{n=1}^{\infty}\frac{\sin(6n)}{n^3}$

___3. $\sum_{n=1}^{\infty}(-1)^n\frac{n!}{2^n}$

___4. $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{5n+6}$

___5. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n\ln n}$

___6. $\sum_{n=1}^{\infty}\frac{(-4)^n}{n^2}$

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2. (3 pts) UNCC1242/EssentialCalculus-Stewart-Sec8.4.Extra2.pg

NOTE: Only 2 attempts are allowed for the whole problem.

Select the FIRST correct reason why the given series converges.

A. Convergent geometric series
B. Convergent p series
C. Comparison (or Limit Comparison) with a geometric or p series
D. Alternating Series Test
E. Ratio Test

___1. $\sum_{n=1}^{\infty}\frac{6(4)^n}{7^{2n}}$

___2. $\sum_{n=1}^{\infty}\frac{n^2}{n^4-3}$

___3. $\sum_{n=1}^{\infty}\frac{(n+1)}{7^{2n}}$

___4. $\sum_{n=1}^{\infty}\frac{(-1)^n}{n^4}$

___5. $\sum_{n=1}^{\infty}\frac{(-1)^n}{6n+2}$

___6. $\sum_{n=1}^{\infty}\frac{\sin^2(3n)}{n^2}$

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3. (3 pts) UNCC1242/EssentialCalculus-Stewart-Sec8.4.Extra3.pg

NOTE: Only 2 attempts are allowed for the whole problem.

Select the FIRST correct reason why the given series converges.

A. Convergent geometric series
B. Convergent p-series
C. Integral test
D. Comparison with a convergent p-series
E. Converges by limit comparison test
F. Converges by alternating series test

___1. $\sum_{n=1}^{\infty}\frac{(-1)^n}{\ln(5n)}$

___2. $\sum_{n=1}^{\infty}ne^{-n^2}$

___3. $\sum_{n=1}^{\infty}\left(\frac{-e}{\pi}\right)^n$

___4. $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n^4-3}$

___5. $\sum_{n=1}^{\infty}\frac{\sin^2(5n)}{n^2}$

___6. $\sum_{n=2}^{\infty}\frac{4}{n(\ln(n))^2}$

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4. (3 pts) UNCC1242/EssentialCalculus-Stewart-Sec8.4.Extra4.pg

NOTE: Only 2 attempts are allowed for the whole problem.

Select the FIRST correct reason why the given series diverges.

A. Diverges because the terms don't have limit zero
B. Divergent geometric series
C. Divergent p series
D. Integral test
E. Comparison with a divergent p series
F. Diverges by limit comparison test
G. Diverges by alternating series test

___1. $\sum_{n=1}^{\infty}\frac{1}{\sqrt{n}+6}$

___2. $\sum_{n=1}^{\infty}(n)^{-\frac{1}{5}}$

___3. $\sum_{n=1}^{\infty} \frac{1}{n\ln(n)}$

___4. $\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{\ln(4)}$

___5. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}-8}$

___6. $\sum_{n=1}^{\infty} \frac{5n+6}{(-1)^n}$

5. (3 pts) UNCC1242/EssentialCalculus-Stewart-Sec8.4.Extra5.pg

NOTE: Only 2 attempts are allowed for the whole problem.

For each of the series below select the letter from A to C that best applies and the letter from D to K that best applies. A possible answer is AF, for example.

A. The series is absolutely convergent.
B. The series is conditionally convergent.
C. The series diverges.

D. The alternating series test shows the series converges.
E. The series is a p-series.
F. The series is a geometric series.
G. By comparison with a p-series.
H. By comparison with a geometric series.
I. Partial sums of the series telescope.
J. The terms of the series do not have limit zero.
K. By the ratio test.

___1. $\sum_{n=1}^{\infty} \frac{1}{5^n+9}$

___2. $\sum_{n=1}^{\infty} \frac{(-5)^n}{n!}$

___3. $\sum_{n=1}^{\infty} \frac{1}{n\sqrt{n}}$

___4. $\sum_{n=2}^{\infty} \frac{(-1)^n}{9^n n!}$

___5. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\pi}$

___6. $\sum_{n=1}^{\infty} \frac{2+\sqrt{n}}{n}$

6. (3 pts) UNCC1242/EssentialCalculus-Stewart-Sec8.4.Extra6.pg

NOTE: Only 2 attempts are allowed for the whole problem.

Determine whether each series converges or not. For the series which converges, enter the sum of the series. For the series which diverges enter "DIV" (without quotes).

1. $\sum_{n=1}^{\infty} \frac{7^n}{6^n} =$ ________,

2. $\sum_{n=0}^{\infty} \frac{3^n}{7^{2n+1}} =$ ________,

3. $\sum_{n=1}^{\infty} \frac{7}{n(n+1)} =$ ________,

4. $\sum_{n=5}^{\infty} \frac{6^n}{7^n} =$ ________,

5. $\sum_{n=1}^{\infty} \frac{9^n}{9^{n+4}} =$ ________,

6. $\sum_{n=1}^{\infty} \frac{6^n+3^n}{7^n} =$ ________.

Webwork: Series practice

№1 $\sum (-1)^{n+1} \frac{4^n}{n!}$, $\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \left|\frac{4^{n+1}}{(n+1)!} \cdot \frac{n!}{4^n}\right| = \lim \left|\frac{4}{n+1}\right| = 0 < 1$

A

the series is ~~always~~ absolutely convergent

$\sum \frac{\sin(6n)}{n^3}$, $\left|\frac{\sin 6n}{n^3}\right| < \frac{1}{n^3}$, $\sum \frac{1}{n^3}$ is a conv. p-series

A

$\Rightarrow \sum \frac{\sin 6n}{n^3}$ is absolutely conv.

$\sum (-1)^n \frac{n!}{2^n}$, $\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim \left|\frac{(n+1)!}{2^{n+1}} \cdot \frac{2^n}{n!}\right| = \lim_{n\to\infty} \left|\frac{n+1}{2}\right| = \infty > 1$

D

$\Rightarrow$ the series is ~~abs.~~ divergent

$\sum \frac{(-1)^{n+1}}{5n+6}$, $\lim b_n = \lim_{n\to\infty} \frac{1}{5n+6} = 0$ and the b_n's are decreasing

C

$\Rightarrow$ the series is convergent.

Is it abs. conv.??

$\sum \frac{1}{5n+6}$ is div by limiting comparison with

$\sum \frac{1}{n}$ (div harmonic)

so $\sum \frac{(-1)^{n+1}}{5n+6}$ is cond. conv.

$\sum \frac{(-1)^n}{n \ln n}$ conv. by alt. series test

C

but $\sum \frac{1}{n \ln n}$ is not conv. by the integral test since $\int_1^\infty \frac{1}{x \ln x} dx$

$= \ln(\ln x)\Big|_1^\infty = \infty$

so $\sum \frac{(-1)^n}{n \ln n}$ is conv. but not absolutely conv.

$\Rightarrow$ conditionally conv.

$\frac{(-4)^n}{n^2}$, ratio test $\lim \left| \frac{\frac{(-4)^{n+1}}{(n+1)^2}}{\frac{(-4)^n}{n^2}} \right| = \lim \left| \frac{4^{n+1}}{(n+1)^2} \cdot \frac{n^2}{4^n} \right|$ (2)

D

$= \lim \left| 4 \cdot \frac{n^2}{(n+1)^2} \right| = 4 > 1$ Div.

$N^o 2$ • $\sum \frac{6(4)^n}{7^{2n}}$ geo $r = \frac{4}{7^2} = \frac{4}{49} < 1 \Rightarrow$ conv A

• $\sum \frac{n^2}{n^4 - 3}$ limit comparison with $\sum \frac{1}{n^2}$ conv. C

$\lim \frac{\frac{n^2}{n^4-3}}{\frac{1}{n^2}} = \frac{n^4}{n^4-3} = 1 \neq 0, \neq \infty$

• $\sum \frac{n+1}{7^{2n}}$, Ratio test, $\lim \left| \frac{\frac{n+2}{7^{2(n+1)}}}{\frac{n+1}{7^{2n}}} \right| = \lim \left| \frac{n+2}{7^{2n+2}} \cdot \frac{7^{2n}}{n+1} \right|$

E

$= \lim \left| \frac{1}{7^2} \cdot \frac{n+2}{n+1} \right| = \frac{1}{49} < 1$ Conv.

• $\sum \frac{(-1)^n}{n^4}$ conv. AST D

• $\sum \frac{(-1)^n}{6n+2}$ // D

• $\sum \frac{\sin^2(3n)}{n^2} < \sum \frac{1}{n^2}$ conv. C

3\. $\bullet\ \sum \frac{(-1)^n}{\ln(5n)}$ conv. AST (F) ③

because the terms $b_n = \frac{1}{\ln(5n)}$ are decreasing with limit 0

$\bullet\ \sum n e^{-n^2}$ Integral test $\int_1^\infty x e^{-x^2} dx$ Let $u = -x^2$, $du = -2x\,dx$

(C)

$= \int \frac{e^u}{-2} du = -\frac{e^{-x^2}}{2}\Big|_1^\infty = -\frac{e^{-\infty}}{2} + \frac{e^{-1}}{2}$

$= \frac{e^{-1}}{2}$

Conv.

$\bullet\ \sum \left(\frac{-e}{\pi}\right)^n$ geo $|r| = \left|\frac{-e}{\pi}\right| < 1$ conv.

(A)

$\bullet\ \sum \frac{\sqrt{n}}{n^4 - 3}$, limit comparison with $\sum \frac{1}{n^{3.5}}$ conv p-series $p = 3.5 > 1$

because comparison doesn't work.

(E)

$\lim \frac{\frac{\sqrt{n}}{n^4-3}}{\frac{1}{n^{3.5}}} = \lim \frac{n^4}{n^4 - 3} = 1 \neq 0, \neq \infty$ both series behave the same.

$\bullet\ \sum \frac{\sin^2(5n)}{n^2} < \sum \frac{1}{n^2}$ conv p series, $p = 2 > 1$

(D)

$\bullet\ \sum_{n=2}^{\infty} \frac{4}{n(\ln(n))^2}$ Let $u = \ln x \Rightarrow du = \frac{1}{x} dx$

$\int_2^\infty \frac{4}{x(\ln x)^2} dx = 4 \int \frac{1}{u^2} du = \frac{-4}{u} \Big|$

Conv.

(C)

$= -\frac{4}{\ln x}\Big|_2^\infty = \frac{-4}{\ln\infty} - \frac{-4}{\ln 2} = \frac{4}{\ln 2}$

$\frac{-4}{\ln\infty} = 0$

series practice web #4] F

- $\sum \frac{1}{\sqrt{n}+6}$, $\frac{1}{\sqrt{n}+6} < \frac{1}{\sqrt{n}}$

$\sum \frac{1}{\sqrt{n}}$ is div p-series

Direct comparison doesn't work

go with $\boxed{\text{limit comparison}}$

$\lim \frac{\frac{1}{\sqrt{n}+6}}{\frac{1}{\sqrt{n}}} = \lim \frac{\sqrt{n}}{\sqrt{n}+6} = 1 \neq 0, \neq \infty$ both series behave the same

- $\sum_{n=1}^{\infty} n^{-1/5} = \sum \frac{1}{n^{1/5}}$ p-series $p = \frac{1}{5} \leq 1$ Divergent.

C

- $\sum_{n=1}^{\infty} \frac{1}{n \ln n}$, $\int \frac{1}{x \ln x} dx = \int \frac{du}{u} = \ln(u) = \ln(\ln x)\Big|_1^{\infty}$

let $u = \ln x \Rightarrow du = \frac{1}{x} dx$ $= \ln(\ln \infty) - \ln(\ln 1)$

∞

Div.

D

- $\sum \frac{\cos(n\pi)}{\ln(4)} = \sum \frac{(-1)^n}{\ln 4}$, $\lim \frac{(-1)^n}{\ln 4} = \frac{1}{\ln 4}$ or $\frac{-1}{\ln 4}$

A Divergent by the test for divergence $\neq 0$

- $\sum \frac{1}{\sqrt{n}-8}$, $\frac{1}{\sqrt{n}-8} > \frac{1}{\sqrt{n}}$, $\sum \frac{1}{\sqrt{n}}$ Div

E Divergent by direct comparison test

- $\sum_{n=1}^{\infty} \frac{5n+6}{(-1)^n}$, terms are not going to zero

test for divergence.

A

Series practice

#5) $\sum \frac{1}{5^n+9}$, $\frac{1}{5^n+9} < \frac{1}{5^n}$, $\sum \frac{1}{5^n}$ is conv. geo $r=\frac{1}{5}<1$

↳ series is also conv. by comparison (H)

$\sum \left|\frac{1}{5^n+9}\right| = \sum \frac{1}{5^n+9}$. so series is abs. conv. (A)

$\sum \frac{(-5)^n}{n!}$, Ratio test: $\lim \left| \frac{\frac{(-5)^{n+1}}{(n+1)!}}{\frac{(-5)^n}{n!}} \right| = \lim \left| \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} \right|$

(A) (K)

$= \lim \left| \frac{5^{n+1}}{5^n} \cdot \frac{n!}{(n+1)!} \right| = \lim \left| 5 \cdot \underbrace{\frac{1}{n+1}}_{\to 0} \right| =$

$= 0 < 1$

Series is Abs. conv.

$\sum \frac{1}{n\sqrt{n}} = \sum \frac{1}{n^{1.5}}$ conv. ~~p~~-series (E)

$p = 1.5 > 1$

$\sum \left| \frac{1}{n\sqrt{n}} \right| = \sum \frac{1}{n^{1.5}}$. So series is abs. conv. (A)

$\sum \frac{(-1)^n}{9^n n!}$ AST, terms are decreasing to 0

Series is convergent.

$\sum \left| \frac{(-1)^n}{9^n n!} \right| = \sum \frac{1}{9^n n!} < \sum \frac{1}{9^n}$ $r=\frac{1}{9}<1$

↳ conv. by comparison to a geo series

Abs. convergent (A) (H)

#5 contd | $\sum \frac{(-1)^n}{n\pi}$ AST, convergent because terms are ↓ to 0

$\sum \left|\frac{(-1)^n}{n\pi}\right| = \sum \frac{1}{n\pi} = \frac{1}{\pi}\sum \frac{1}{n}$ DiV.

series is not absolutely convergen

Cond. Convergent.

B D

$\sum \frac{2+\sqrt{n}}{n}$ $\qquad \frac{2+\sqrt{n}}{n} > \frac{\sqrt{n}}{n} = \frac{1}{\sqrt{n}}$

↳ series is divergent

$\sum \frac{1}{\sqrt{n}}$ is div p-series, $p=\frac{1}{2}\le 1$

C H

#6 | $\sum \frac{7^n}{6^n}$, $r=\frac{7}{6}>1 \to \boxed{\text{DiV}}$ geo

$\sum_{n=0}^{\infty} \frac{3^n}{7^{2n+1}}$, $r=\frac{3}{7^2}=\frac{3}{49}<1 \Rightarrow$ conv. $S=\frac{a}{1-r}=\frac{\frac{3^0}{7^{0+1}}}{1-\frac{3}{49}}=\frac{\frac{1}{7}}{\frac{46}{49}}$

$=\boxed{\frac{7}{46}}$

$\sum_{n=5}^{\infty} \frac{6^n}{7^n}$, $r=\frac{6}{7}<1$ conv, $S=\frac{a}{1-r}=\frac{\frac{6^5}{7^5}}{1-\frac{6}{7}}=\frac{\frac{6^5}{7^5}}{\frac{1}{7}}=\boxed{\frac{6^5}{7^6}}$

$\sum \frac{9^n}{9^{n+4}} = \sum \frac{9^n}{9^n 9^4} = \sum \frac{1}{9^4}$, $\boxed{\text{div}}$ by test for divergen

limit of terms $= \frac{1}{9^4} \ne 0$

$\sum_{n=1}^{\infty} \frac{6^n+3^n}{7^n} = \sum \left(\frac{6}{7}\right)^n + \left(\frac{3}{7}\right)^n$ conv, Sum $= \frac{\frac{6}{7}}{1-\frac{6}{7}} + \frac{\frac{3}{7}}{1-\frac{3}{7}} = 6+\frac{3}{4} = \boxed{6.75}$

$\sum \frac{1}{n(n+1)} = \sum \frac{1}{n} - \frac{1}{n+1}$ Telescoping series (after doing $\frac{A}{n}+\frac{B}{n+1}\dots$)

$S_n = \left(\frac{1}{1}-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\dots+\left(\frac{1}{n}-\frac{1}{n+1}\right) = 1-\frac{1}{n+1}$

$\sum = \lim S_n = \boxed{1}$