

Solution to #12-14-15/
Section 5.4

Webwork Section 5.4 #12 Book Problem 23

$$F'(x) = f(x) \Rightarrow F''(x) = f'(x)$$

need to find $f'(x)$

If $F(x) = \int_{11}^x f(t) dt$, where $f(t) = \int_1^{t^5} \frac{\sqrt{6+u^2}}{u} du$, find: $f'(t) = \frac{\sqrt{6+(t^5)^2}}{t^5} \cdot 5t^4$

$F''(x) = \boxed{}$ and $F''(3) = \boxed{}$

$$= \frac{5\sqrt{6+x^{10}}}{x}$$

$$= \frac{5\sqrt{6+3^{10}}}{3} = \dots$$

$$= \frac{\sqrt{6+t^{10}}}{t^5} \cdot 5t^4$$

$$= \frac{5\sqrt{6+t^{10}}}{t}$$

so $F''(x) = f'(x) = \frac{5\sqrt{6+x^{10}}}{x}$

Webwork 5.4 #14 Book Problem 31

Find a function f and a positive number a such that

$$\left(1 + \int_a^x \frac{f(t)}{t^6} dt = 6x^{-3}\right)' \Rightarrow \frac{f(x)}{x^6} = -18x^{-4}$$

$f(x) = \boxed{-18x^2}$

$a = \boxed{\sqrt[3]{6}}$

$$\Rightarrow f(x) = -18x^{-4}(x^6)$$

$f(x) = -18x^2$
plug in eq.

$$1 + \int_a^x \frac{-18t^2}{t^6} dt = 6x^{-3}$$

$$1 + \int_a^x -18t^{-4} dt = 6x^{-3} \Rightarrow 1 + \frac{-18t^{-3}}{-3} \Big|_a^x = 6x^{-3}$$

$$\Rightarrow 1 + \frac{6}{x^3} - \frac{6}{a^3} = 6x^{-3}$$

$$\Rightarrow 1 + \frac{6}{x^3} - \frac{6}{a^3} = 6x^{-3}$$

$$\Rightarrow 1 - \frac{6}{a^3} = 6x^{-3} - \frac{6}{x^3} = 0$$

$$\Rightarrow 1 = \frac{6}{a^3} \Rightarrow a^3 = 6$$

$$\Rightarrow \boxed{a = \sqrt[3]{6}}$$

Webwork #15 Extra Problem 1
(5.4)

Consider the function

$$f(x) = \begin{cases} x & \text{if } x < 1 \\ \frac{1}{x} & \text{if } x \geq 1 \end{cases}$$

Evaluate the definite integral $\int_{-4}^3 f(x) dx = \boxed{-7.5 + \ln 3}$

$$\int_{-4}^3 f(x) dx = \int_{-4}^1 x dx + \int_1^3 \frac{1}{x} dx$$

$$= \frac{x^2}{2} \Big|_{-4}^1 + \ln|x| \Big|_1^3 = \frac{1}{2} - \frac{16}{2} + \ln 3 - \ln 1 = \boxed{-\frac{15}{2} + \ln 3}$$