int by parts Rules | $\int x^n e^x dx$ or $\int x^n \cos x dx$ or $\int x \sin x dx$, pick $u = x$

$\int x \ln x dx$ or $\int x \arctan x dx$ or $\int x \arcsin x dx$, pick $u = \ln x$ or arctan or arcsin

UNCC1242/EssentialCalculus-Stewart-Sec6.1.1.pg

$\int u dv = uv - \int v du$

1) **Book Problem 1**

Use integration by parts to evaluate the integral $\int x\ln(5x)dx =$ $= uv - \int v du$ $+C$

Let $u = \ln(5x) \rightarrow du = \frac{5}{5x} dx = \frac{1}{x} dx$

Let $dv = x dx \rightarrow v = \frac{x^2}{2}$

$= \ln(5x)\frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x} dx$

$= \frac{x^2 \ln(5x)}{2} - \frac{1}{2}\int x dx$

$= \frac{x^2 \ln(5x)}{2} - \frac{1}{2} \cdot \frac{x^2}{2} + C$

$= \boxed{\frac{x^2 \ln(5x)}{2} - \frac{x^2}{4} + C}$

2) **Book Problem 3**

Use integration by parts to evaluate the integral $\int x\sin(9x)dx =$ $+C$

Let $u = x \rightarrow du = dx$

$dv = \sin(9x)dx \rightarrow v = -\frac{\cos(9x)}{9}$

$\int x \sin(9x) dx = uv - \int v du$

$= x\left(-\frac{\cos(9x)}{9}\right) - \int -\frac{\cos(9x)}{9} dx$

$= -\frac{x\cos(9x)}{9} + \frac{1}{9}\int \cos(9x) dx$

$=$ " $+ \frac{1}{9} \frac{\sin(9x)}{9} + C$

$= \boxed{-\frac{x\cos(9x)}{9} + \frac{\sin(9x)}{81} + C}$

3) **Book Problem 5**

Use integration by parts to evaluate the integral $\int r e^{r/8} dr =$

Let $u = r \rightarrow du = dr$

$dv = e^{r/8} dr \rightarrow v = \frac{e^{r/8}}{1/8}$

$v = 8e^{r/8}$

$\int r e^{r/8} dr = r \, 8e^{r/8} - \int 8e^{r/8} dr$

$= 8re^{r/8} - 8\int e^{r/8} dr$

$=$ " $- 8 \frac{e^{r/8}}{1/8} + C$

$= \boxed{8re^{r/8} - 64e^{r/8} + C}$

Rule: $\int e^{ax} dx = \frac{e^{ax}}{a} + C$

$\int \cos(ax) dx = \frac{\sin(ax)}{a} + C$

$\int$ . . . .

4)

Book Problem 7

Use integration by parts to evaluate the integral $\int x^2 \sin(8x)dx = $ $x^2\left(-\frac{\cos(8x)}{8}\right) - \int -\frac{\cos(8x)}{8} \cdot 2x\,dx$

Let $u = x^2 \longrightarrow du = 2x\,dx$

$dv = \sin(8x)dx \longrightarrow v = -\frac{\cos(8x)}{8}$

$= -\frac{x^2\cos(8x)}{8} + \frac{1}{4}\int x\cos(8x)\,dx$ (another int. by parts)

Let $u = x \longrightarrow du = dx$

$dv = \cos(8x)dx \longrightarrow v = \frac{\sin(8x)}{8}$

$= \;'' \; + \frac{1}{4}\left[\frac{x\sin(8x)}{8} - \int \frac{\sin(8x)}{8}dx\right]$

$= \;'' \; + \frac{1}{4}\left[\frac{x\sin(8x)}{8} + \frac{\cos(8x)}{64}\right] + C = \boxed{-\frac{x^2\cos(8x)}{8} + \frac{x\sin(8x)}{32} + \frac{\cos(8x)}{256} + C}$

5)

Book Problem 9

Use integration by parts to evaluate the integral $\int \ln(7x+5)dx = $ +C

Let $u = \ln(7x+5) \longrightarrow du = \frac{7}{7x+5}dx$

$dv = dx \longrightarrow v = x$

$\boxed{\int \frac{1}{7x+5}dx = \frac{\ln|7x+5|}{7} + C}$

$\int \ln(7x+5)dx = uv - \int v\,du$

$= \ln(7x+5)x - \int x \cdot \frac{7}{7x+5}dx$

$= \;'' \; - \int \frac{7x}{7x+5}dx$

$= \;'' \; - \int\left(\frac{7x+5}{7x+5} - \frac{5}{7x+5}\right)dx$

$= \;'' \; - \int\left(1 - \frac{5}{7x+5}\right)dx$

$\boxed{= x\ln(7x+5) - \left[x - \frac{5\ln|7x+5|}{7}\right] + C}$

6)

Book Problem 11

Use integration by parts to evaluate the integral $\int \arctan(6t)dt = $ +C

Let $u = \arctan(6t) \longrightarrow du = \frac{6}{1+36t^2}dt$

$dv = dt \longrightarrow v = t$

$\int \arctan(6t)dt = uv - \int v\,du = t\arctan(6t) - \int \frac{t \cdot 6}{1+36t^2}dt$

$= t\arctan(6t) - \int \frac{6t}{w} \cdot \frac{dw}{72t}$ ⟵ use substitution

Let $w = 1+36t^2$

$dw = 72t\,dt$

$= t\arctan(6t) - \frac{1}{12}\ln|w| + C$

7)

Book Problem 13

Use integration by parts twice to evaluate $\int e^{4t}\cos(7t)dt$:

Step 1: Let $u=e^{4t}$ and $dv=\cos(7t)dt$. Apply integration by parts to get a result of the form

$u=e^{4t} \rightarrow du=4e^{4t}dt$

$dv=\cos(7t)dt \rightarrow v=\frac{\sin(7t)}{7}$

$\int e^{4t}\cos(7t)dt = uv-\int v\,du = e^{4t}\left(\frac{\sin 7t}{7}\right) - \int \frac{\sin(7t)}{7}4e^{4t}dt$

$\int e^{4t}\cos(7t)dt = \frac{e^{4t}\sin(7t)}{7} + \int -\frac{4}{7}e^{4t}\sin(7t)\,dt.$

Step 2: Apply integration by parts once again, letting $u=e^{4t}$ and identifying dv to get a result of the form below

Let $u=e^{4t} \rightarrow du=4e^{4t}dt$

$dv=\sin(7t)dt \rightarrow v=-\frac{\cos(7t)}{7}$

so $\int e^{4t}\sin(7t)dt = e^{4t}\left(-\frac{\cos 7t}{7}\right) - \int -\frac{\cos(7t)}{7}4e^{4t}dt$

$= -\frac{e^{4t}\cos(7t)}{7} + \frac{4}{7}\int e^{4t}\cos(7t)dt$

so $\int e^{4t}\cos(7t)dt = \frac{e^{4t}\sin(7t)}{7} - \frac{4}{7}\int e^{4t}\sin(7t)dt = \frac{e^{4t}\sin(7t)}{7} - \frac{4}{7}\left(-\frac{e^{4t}\cos 7t}{7} + \frac{4}{7}\int e^{4t}\cos(7t)dt\right)$

$\int e^{4t}\cos(7t)dt = \frac{e^{4t}\sin 7t}{7} + \frac{4}{49}e^{4t}\cos 7t - K\int e^{4t}\cos(7t)dt$, where $K=\frac{16}{49}$, a positive number.

$\Rightarrow \int e^{4t}\cos(7t)dt + \frac{16}{49}\int e^{4t}\cos(7t)dt = \frac{e^{4t}\sin 7t}{7} + \frac{4}{49}e^{4t}\cos 7t$ (by adding $\frac{16}{49}\int e^{4t}\cos 7t\,dt$ to both sides)

$\Rightarrow \left(1+\frac{16}{49}\right)\int e^{4t}\cos(7t)dt = \text{''} + \text{''}$

$\Rightarrow \left(\frac{65}{49}\right) \text{''} = \text{''} + \text{''}$

$\Rightarrow \int e^{4t}\cos(7t)dt = \left(\frac{49}{65}\right)\left(\frac{e^{4t}\sin 7t}{7} + \frac{4}{49}e^{4t}\cos 7t\right)$ (by dividing both sides by $\frac{65}{49}$)

The wrap up: Adding $K\int e^{4t}\cos(7t)dt$ to both sides of the equation, and dividing by $(K+1)$ yields the answer to the original question: $\int e^{4t}\cos(7t)dt = \left(\frac{49}{65}\right)\left(\frac{e^{4t}\sin(7t)}{7} + \frac{4}{49}e^{4t}\cos(7t)\right) + C.$

let $u = y \rightarrow du = dy$

$dv = e^{-8y}dy \rightarrow v = \frac{e^{-8y}}{-8}$

8 **Extra Problem**

Use integration by parts to evaluate the integral $\int_0^3 \frac{y}{e^{8y}}dy = \int_0^3 y e^{-8y} dy$

$= uv\Big|_0^3 - \int_0^3 v\,du = \ldots$

$= y\frac{e^{-8y}}{-8}\Big|_0^3 - \int_0^3 \frac{e^{-8y}}{-8}dy = \frac{ye^{-8y}}{-8}\Big|_0^3 - \frac{e^{-8y}}{64}\Big|_0^3 = \frac{3e^{-24}}{-8} - 0 - \left(\frac{e^{-24}}{64} - \frac{1}{64}\right)$

$= \left(\frac{-3}{8} - \frac{1}{64}\right)e^{-24} + \frac{1}{64}$

$= \frac{-25}{64}e^{-24} + \frac{1}{64}$

9 **Book Problem 23**

Use a combination of integration by parts and substitution to evaluate the definite integral $\int_0^{1/3} \arcsin(3x)dx$.

let $u = \arcsin(3x) \rightarrow du = \frac{3}{\sqrt{1-9x^2}}dx$

$dv = dx \rightarrow v = x$

get $uv - \int v\,du$

Step 1: Integration by parts, let u= and $dv=dx$.

$\int \arcsin(3x)dx = x\arcsin(3x) - \int \frac{3x}{\sqrt{1-9x^2}}$ dx.

Step 2: To evaluate the remaining indefinite integral, one uses the substitution $u(x)$= $1-9x^2$, leading to the result that $\int \arcsin(3x)dx$= $x\arcsin(3x) + \frac{1}{3}\sqrt{1-9x^2}$ +C.

$du = -18x\,dx \rightarrow dx = \frac{du}{-18x}$

get $\int \frac{3x}{\sqrt{1-9x^2}}dx = \int \frac{3x}{\sqrt{u}} \cdot \frac{du}{-18x}$

$= -\frac{1}{6}\int u^{-1/2}du$

$= -\frac{1}{6}2u^{1/2} + C$

$= -\frac{1}{3}\sqrt{1-9x^2} + C$

Step 3: Using the above indefinite integral in combination with the Evaluation Theorem, gives the final answer as:

$\int_0^{1/3} \arcsin(3x)dx$= $x\arcsin(3x) + \frac{1}{3}\sqrt{1-9x^2}\Big|_0^{1/3}$

$= \left(\frac{1}{3}\arcsin(1) + \frac{1}{3}\sqrt{1-9\cdot\frac{1}{9}}\right) - \left(0 + \frac{1}{3}\sqrt{1-0}\right)$

$= \frac{1}{3}\cdot\frac{\pi}{2} - \frac{1}{3} = \frac{\pi}{6} - \frac{1}{3}$

10

$\int \sin(4\sqrt{x})\,dx$

$\sin 6$
$w = 4\sqrt{x}$
$\Rightarrow \sqrt{x} = \frac{w}{4}$

Book Problem 27

Use a **combination of substitution and parts** to evaluate the integral $\int \sin(4\sqrt{x})dx$:

$$w = 4\sqrt{x} \Rightarrow dw = 4 \cdot \frac{1}{2} x^{-1/2} dx \Rightarrow dw = \frac{4}{2\sqrt{x}} dx \Rightarrow dx = \frac{2\sqrt{x}}{4} dw$$

$$dx = \frac{1}{2} \cdot \frac{w}{4} dw$$

$$dx = \frac{w}{8} dw$$

Step 1: Substitution, let $w=4\sqrt{x}$. Note: We use w here for the substitution instead of the more common variable u, since it is convenient for us to reserve u and v for the upcoming integration by parts.

It follows that $dw=\frac{4}{2\sqrt{x}}dx$ and $dx=\frac{2\sqrt{x}}{4}dw$. To successfully continue with substitution, it is now necessary to rewrite dx strictly in terms of w. Thus $dx=f(w)dw$, where $f(w)=$ $w/8$

compare with $dx = g(w)dw$

Complete the substitution, to get $\int \sin(4\sqrt{x})dx=\int g(w)dw$ where $g(w)=$ $\frac{w}{8}\sin w$.

$$\int \sin(4\sqrt{x})dx = \int \sin w \cdot \frac{w}{8} dw = \int \frac{w}{8} \sin w\, dw$$ do int. by parts.

This is $= \int g(w)\,dw$ in the problem's notation

Step 2: Use integration by parts to integrate $\int g(w)dw$.

Let $u=$ $\frac{w}{8}$ and $dv=\sin(w)dw$. This gives (as a function of w), $\int g(w)dw=$ $uv - \int v\,du$

$du = \frac{1}{8} dw$ $\quad v = -\cos w$

$$= \frac{w}{8}(-\cos w) - \int -\cos w \frac{1}{8} dw$$

$$= \boxed{-\frac{w \cos w}{8} + \frac{1}{8}\sin w + C}$$

Step 3: Substitute $4\sqrt{x}$ for w, to get $\int \sin(4\sqrt{x})dx=$ $\boxed{-\frac{4\sqrt{x}\cos(4\sqrt{x})}{8} + \frac{1}{8}\sin(4\sqrt{x}) + C}$

11)

Book Problem 29

Evaluate the definite integral $\int_{\sqrt[6]{\pi/2}}^{\sqrt[6]{\pi}} t^{11}\cos(t^6)\,dt = \int t^{11}\cos w \,\frac{dw}{6t^5} = \int \frac{t^6}{6}\cos w\,dw = \int \frac{w}{6}\cos w\,dw$

substitution: let $w = t^6$

$dw = 6t^5\,dt$

$dt = \frac{dw}{6t^5}$

Step 1: Start by making the substitution $w=t^6$. This leads to

$\int_{\sqrt[6]{\pi/2}}^{\sqrt[6]{\pi}} t^{11}\cos(t^6)dt = \int_a^b \frac{w}{6}\cos w \, dw$ where $a=\frac{\pi}{2}$ and $b=\pi$.

lower limit

if $t = \sqrt[6]{\frac{\pi}{2}} \Rightarrow w = \left(\sqrt[6]{\frac{\pi}{2}}\right)^6 = \frac{\pi}{2} = a$

upper limit

if $t = \sqrt[6]{\pi} \Rightarrow w = (\sqrt[6]{\pi})^6 = \pi = b$

Step 2: Use integration by parts to evaluate the new integral.

This give the value of the original definite integral to be: ______.

need to calculate $\int_{\frac{\pi}{2}}^{\pi} \frac{w}{6}\cos w\,dw$ by integration by parts

let $u = \frac{w}{6} \rightarrow du = \frac{1}{6}dw$

$dv = \cos w\,dw \rightarrow v = \sin w$

$= uv\Big|_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} v\,du$

$= \frac{w\sin w}{6}\Big|_{\pi/2}^{\pi} - \int_{\pi/2}^{\pi} \sin w \cdot \frac{1}{6}dw$

$= \frac{w\sin w}{6}\Big|_{\pi/2}^{\pi} + \frac{\cos w}{6}\Big|_{\frac{\pi}{2}}^{\pi}$

$= \frac{\pi\sin\pi}{6} - \frac{\pi/2\,\sin\pi/2}{6} + \frac{\cos\pi}{6} - \frac{\cos\pi/2}{6}$

$= 0 - \frac{\pi/2}{6} + \frac{-1}{6} - 0 = \boxed{-\frac{1}{6} - \frac{\pi}{12}}$

12)

Book Problem 35

Use integration by parts to establish the reduction formula: $\int(\ln x)^n\,dx = x(\ln x)^n - n\int(\ln x)^{n-1}\,dx$:

To begin the integration by parts, let $u=$ $(\ln x)^n$ and $dv=$ 1 dx.

Evaluating $du=$ $n(\ln x)^{n-1}\cdot\frac{1}{x}$ dx and $v=$ x leads immediately to the above reduction formula.

$$\int(\ln x)^n dx = uv - \int v\,du = x(\ln x)^n - \int \cancel{x}\, n(\ln x)^{n-1}\frac{1}{\cancel{x}}\,dx$$
$$= x(\ln x)^n - n\int(\ln x)^{n-1}dx$$

Now apply the reduction formula to the problem: $\int(\ln x)^5 dx$. Getting the final result would be tedious, requiring 5applications of the reduction formula.

$$\int(\ln x)^5 dx = x(\ln x)^5 - 5\int(\ln x)^4 dx$$

So for simplicity just give the result of applying the reduction formula one time. $\int(\ln x)^5\,dx$ is simplified to $x(\ln x)5$ $+\int$ $-5(\ln x)4$ $dx + C$.

13)

Book Problem 39

Use the reduction formula: $\int(\ln x)^n dx=x(\ln x)^n-n\int(\ln x)^{n-1} dx$ to evaluate $\int(\ln x)^3 dx$. To achieve this, you will need to apply the reduction formula 3 times.

First application of the reduction formula (n=3):

$\int(\ln x)^3 dx=$ $x(\ln x)^3$ $+\int$ $-3(\ln x)^2$ dx.

$= \;'' \; -3\int(\ln x)^2 dx = x(\ln x)^3 - 3\left[x(\ln x)^2 - 2\int \ln x\, dx\right]$

Second application of the reduction formula (n=2):

$\int(\ln x)^3 dx=$ $x(\ln x)^3 - 3x(\ln x)^2 + \int 6 \ln x$ dx.

if $n=1$: $\int \ln x\,dx = x\ln x - 1\int(\ln x)^0 dx$

$= x\ln x - \int dx$

$= x\ln x - x + C$

Third application of the reduction formula: (n=1)

$\int(\ln x)^3 dx=$ $x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x$ $+\int$ -6 dx.

Wrap-up: So completing the final integration above,

$\int(\ln x)^3 dx=$ $'' - '' + '' \quad - 6x$ $+C$.

$\int(\ln x)^3 dx = x(\ln x)^3 - 3x(\ln x)^2 + 6x\ln x - 6x + C$

14)

Book Problem 43

A particle that moves along a straight line has velocity $v(t)=t^2 e^{-8t}$ m/s after t seconds. This problem involves determining the distance $x(t)$ that it will travel during the first t seconds.

$\begin{cases} \text{let } u=t^2 \rightarrow du=2t\,dt \\ dv=e^{-8t}dt \rightarrow v=\frac{e^{-8t}}{-8} \end{cases}$ $\quad uv-\int v\,du = t^2\frac{e^{-8t}}{-8} - \int \frac{e^{-8t}}{-8} \cdot 2t\,dt$

Step 1. Use integration by parts once with $u=t^2$ and $dv=e^{-8t}\,dt$ to begin determining the indefinite integral (antiderivative) of t^2e^{-8t}. This gives

another int by parts $uv - \int v\,du$

$x(t)=\int t^2e^{-8t}\,dt= \boxed{\frac{t^2e^{-8t}}{-8}} + \int \boxed{\frac{1}{4}te^{-8t}}\,dt. = \frac{t^2e^{-8t}}{-8} + \left[\frac{1}{4}t\cdot\frac{e^{-8t}}{-8} - \int \frac{e^{-8t}}{-8}\cdot\frac{1}{4}dt\right.$

do another int by parts let $u=\frac{1}{4}t \Rightarrow du=\frac{1}{4}dt$, and $dv=e^{-8t}dt \Rightarrow v=\frac{e^{-8t}}{-8}$ $\quad = '' + '' + \frac{1}{32}\int e^{-8t}dt$

Step 2. Use integration by parts again to complete finding the indefinite integral (antiderivative) of t^2e^{-8t}. This gives

$x(t)=\int t^2e^{-8t}\,dt= \boxed{\frac{t^2e^{-8t}}{-8} - \frac{te^{-8t}}{32} - \frac{e^{-8t}}{256}} +C.$ $\quad = '' + '' + \frac{1}{32}\frac{e^{-8t}}{-8} + C$

Step 3. Use the initial condition (IC) that $x(0)=0$ to determine the value of the constant C: $\boxed{1/256}$

replace $t=0$ in the above equation of x.

$x(0)=0=\frac{0^2}{-8}-\frac{0}{32}-\frac{e^0}{256}+C \Rightarrow C=\frac{1}{256}$

Step 4. Combine the results of steps 2 and 3 above to determine the distance the particle will travel during the first t seconds: $x(t)= \boxed{-e^{-8t}\left(\frac{t^2}{8}+\frac{t}{32}+\frac{1}{256}\right)+\frac{1}{256}}$

15)

Book Problem 45

Suppose that $f(3)=6, f(9)=9, f'(3)=8, f'(9)=5$ and f'' is continuous.

Find the value of the definite integral: $\int_3^9 xf''(x)dx=$ ______ .

$$\int_3^9 x f''(x)\,dx = uv - \int v\,du = x f'(x)\Big|_3^9 - \int_3^9 f'(x)\,dx$$

int. by parts

Let $u = x \longrightarrow du = dx$

$dv = f''(x)dx \longrightarrow v = f'(x)$

$$= [9f'(9) - 3f'(3)] - f(x)\Big|_3^9$$

$$= [\quad '' \quad] - [f(9) - f(3)]$$

$$= [9 * 5 - 3 * 8] - [9 - 6]$$

$$= \ldots$$