1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.1.pg

Rule: $\dfrac{\cdots}{(x+a)(x+b)} = \dfrac{A}{x+a} + \dfrac{B}{x+b}$

$\dfrac{\cdots}{(x+a)(x^2+b)} = \dfrac{A}{x+a} + \dfrac{Bx+C}{x^2+b}$

$\dfrac{\cdots}{(x+a)(x+b)^3} = \cancel{\dfrac{}{x+a}} + \cancel{\dfrac{}{(x+b)}} + \cancel{\dfrac{}{(x+b)^2}} + \cancel{\dfrac{}{x+b}}$

$= \dfrac{A}{x+a} + \dfrac{B}{x+b} + \dfrac{C}{(x+b)^2} + \dfrac{D}{(x+b)^3}$

Book Problem 1

Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the rational functions below. You must get all of the answers correct to receive credit.

(a) $f(x) = \dfrac{8x+1}{(5x+9)(x-5)}$. $= \dfrac{A}{5x+9} + \dfrac{B}{x-5}$

F 1. $\frac{Bx}{5x+9}$
F 2. $\frac{Ex+F}{5x+9}$
F 3. $\frac{Cx+D}{x-5}$
T 4. $\frac{G}{5x+9}$
T 5. $\frac{A}{x-5}$

(b) $g(x) = \dfrac{x+9}{x^3+10x^2+25x}$ $= \dfrac{}{x(x^2+10x+25)} = \dfrac{}{x(x+5)^2} =$

$= \dfrac{A}{x} + \dfrac{B}{(x+5)} + \dfrac{C}{(x+5)^2}$

F 1. $\frac{Cx+D}{x+25}$
T 2. $\frac{F}{(x+5)^2}$
T 3. $\frac{E}{x+5}$
F 4. $\frac{B}{x^2}$
F 5. $\frac{Gx+H}{(x+5)^2}$
T 6. $\frac{A}{x}$

---

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.3.pg

Book Problem 3

Enter a T or an F in each answer space below to indicate whether or not a term of the given type occurs in the general form of the complete partial fractions decomposition of the rational functions below. You must get all of the answers correct to receive credit.

(a) $f(x) = \dfrac{8x+1}{x^2+2x-35}$ $= \dfrac{\cdots}{(x+7)(x-5)} = \dfrac{A}{x+7} + \dfrac{B}{x-5}$

T 1. $\frac{D}{x+7}$
F 2. $\frac{Ex+F}{x^2+2x-35}$
F 3. $\frac{B}{x+5}$
F 4. $\frac{C}{x-7}$
T 5. $\frac{A}{x-5}$

(b) $g(x) = \dfrac{x^2+8}{(x-5)(x^2+x+7)}$ $= \dfrac{A}{x-5} + \dfrac{Bx+C}{x^2+x+7}$

T 1. $\frac{A}{x-5}$
T 2. $\frac{Ex+F}{x^2+x+7}$
F 3. $\frac{Gx^2+H}{x^2+x+7}$
F 4. $\frac{B}{x^2}$
F 5. $\frac{C}{x+7}$
F 6. $\frac{D}{x^2+x+7}$

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.7.pg

Book Problem 7

Consider the indefinite integral $\int \frac{7x^3}{x-2}\,dx$. $= \int \left(7x^2+14x+28+\frac{56}{x-2}\right)dx$

$$\begin{array}{r} 7x^2+14x+28 \\ x-2 \enclose{longdiv}{7x^3} \\ -\left(7x^3-14x^2\right) \\ \hline 14x^2 \\ -\left(14x^2-28x\right) \\ \hline 28x \\ -\left(28x-56\right) \\ \hline 56 \end{array}$$

Using long division, the integrand decomposes into the form

$ax^2+bx+c+\frac{d}{x-2}$ where

$a =$ 7

$b =$ 14

$c =$ 28

$d =$ 56

$= \frac{7x^3}{3}+\frac{14x^2}{2}+28x+56\ln|x-2|+C$

Integrating term by term, we obtain that

$\int \frac{7x^3}{x-2}\,dx =$ ______ $+C$

$\ln(abs(x-2))$

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.9.pg

Book Problem 9

Consider the indefinite integral $\int \frac{7x+17}{x^2+7x+6}\,dx.$ $= \int \frac{7x+17}{(x+6)(x+1)}\,dx$

Using the partial fraction method, the integrand decomposes into the form $\frac{A}{x+1}+\frac{B}{x+6}$ where

$A =$ 2

$B =$ 5

$\frac{7x+17}{(x+6)(x+1)} = \frac{A}{x+1}+\frac{B}{x+6} = \frac{A(x+6)+B(x+1)}{(x+1)(x+6)}$

$\boxed{7x+17 = A(x+6)+B(x+1)}$

Let $x=-1$: $-7+17 = A(5)+B(0) \Rightarrow \boxed{A=2}$

Let $x=-6$: $-42+17 = A(0)+B(-5) \Rightarrow \boxed{B=5}$

Integrating term by term, we obtain that

$\int \frac{7x+17}{x^2+7x+6}\,dx =$ ______ $+C$

$= \int \left(\frac{2}{x+1}+\frac{5}{x+6}\right)dx$

$= \boxed{2\ln|x+1|+5\ln|x+6|+C}$

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.11.pg

Book Problem 11

$$\frac{19}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} \text{ where } A = \_\_\_ \text{ and } B = \_\_\_$$

$\int \frac{19}{x^2-1}dx =$ ______________ +C

$\int_2^6 \frac{19}{x^2-1}dx =$ ________

Recall $\int \frac{1}{u(x)}du = \ln(abs(u(x)))$, not $\ln(u(x))$ which would be incorrect for negative values of $u(x)$.

$$\frac{19}{x^2-1} = \frac{A}{x-1} + \frac{B}{x+1} = \frac{A(x+1)+B(x-1)}{(x-1)(x+1)}$$

$$19 = A(x+1) + B(x-1)$$

Let $x=1$: $19 = 2A + 0B \Rightarrow A = 19/2$

Let $x=-1$: $19 = 0A - 2B \Rightarrow B = -19/2$

$$\int \frac{19}{x^2-1}dx = \int\left(\frac{19/2}{x-1} + \frac{-19/2}{x+1}\right)dx = \boxed{\frac{19}{2}\ln|x-1| - \frac{19}{2}\ln|x+1| + C}$$

$$\int_2^6 \frac{19}{x^2-1}dx = \frac{19}{2}\ln|x-1| - \frac{19}{2}\ln|x+1| \Big|_2^6 = \left(\frac{19}{2}\ln 5 - \frac{19}{2}\ln 7\right) - \left(\frac{19}{2}\ln 1 - \frac{19}{2}\ln 3\right)$$

$$\Rightarrow \boxed{\frac{19}{2}(\ln 5 - \ln 7 + \ln 3)}$$

6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.16.pg

Book Problem 16

Decompose the following fraction using a long division:

$$\frac{3x^3+3x^2-19x-21}{x^2-9} = ax+b+\frac{cx+d}{x^2-9}, \text{ where } = 3x+3+\frac{8x+6}{x^2-9}$$

$a =$ 3

$b =$ 3

$c =$ 8

$d =$ 6

$$\begin{array}{r} 3x+3 \\ x^2-9 \enclose{longdiv}{3x^3+3x^2-19x-21} \\ -(3x^3-27x) \\ \hline 3x^2+8x-21 \\ -(3x^2-27) \\ \hline 8x+6 \end{array}$$

Use the partial fractions method on the resulting fraction to get:

$$\frac{3x^3+3x^2-19x-21}{x^2-9} = ax+b+\frac{A}{x-3}+\frac{B}{x+3}, \text{ where}$$

$A =$ 5

$B =$ 3

Integrate term by term to find

$\int \frac{3x^3+3x^2-19x-21}{x^2-9} =$ ______________ +C

$$\frac{8x+6}{x^2-9} = \frac{8x+6}{(x-3)(x+3)} = \frac{A}{x-3} + \frac{B}{x+3}$$

$$\frac{8x+6}{(x-3)(x+3)} = \frac{A(x+3)+B(x-3)}{(x-3)(x+3)}$$

$$8x+6 = A(x+3)+B(x-3)$$

Let $x=3$: $24+6 = A(6)+B(0) \Rightarrow \boxed{A=5}$

Let $x=-3$: $-24+6 = A(0)+B(-6) \Rightarrow \boxed{B=3}$

$$= \int\left(3x+3+\frac{5}{x-3}+\frac{3}{x+3}\right)dx$$

$$= \boxed{\frac{3x^2}{2} + 3x + 5\ln|x-3| + 3\ln|x+3| + C}$$

7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.17.pg

Book Problem 17

Consider the indefinite integral $\int \frac{5x^2+6x-18}{x(x+3)(x-5)}\,dx = \frac{A}{x} + \frac{B}{x+3} + \frac{C}{x-5} = \frac{A(x+3)(x-5)+Bx(x-5)+Cx(x+3)}{x(x+3)(x-5)}$

Using the partial fraction method, the integrand decomposes into the form

$5x^2+6x-18 = A(x+3)(x-5)+Bx(x-5)+Cx(x+3)$

$\frac{A}{x}+\frac{B}{x+3}+\frac{C}{x-5}$ where $A =$ ___, $B =$ ___ and $C =$ ___.

let $x=0$: $-18 = -15A + 0B + 0C \Rightarrow A = \frac{18}{15} = \frac{6}{5}$

Integrating term by term, we obtain that $\int \frac{5x^2+6x-18}{x(x+3)(x-5)}\,dx =$ ______________ $+C$

let $x=-3$: $45-18-18 = 0A + 24B + 0C \Rightarrow B = \frac{9}{24} = \frac{3}{8}$

$$= \int \left(\frac{6/5}{x} + \frac{3/8}{x+3} + \frac{\cdots}{x-5} \right) dx$$

$$= \frac{6}{5}\ln|x| + \frac{3}{8}\ln|x+3| + \ldots \ln|x-5| + C$$

$$\boxed{\text{Rule: } \int \frac{1}{x^2+a^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right)+C}$$

8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.Extra1.pg

Extra Problem 1

Evaluate the integral $\int \frac{7x-6}{x^2+16}\,dx =$ ______________ $+C$

$$\int \frac{7x-6}{x^2+16}\,dx = \int \frac{7x}{x^2+16}\,dx - \int \frac{6}{x^2+16}\,dx$$

let $u = x^2+16$

$du = 2x\,dx$

$dx = \frac{du}{2x}$

$$= \int \frac{7x}{u}\cdot\frac{du}{2x} - 6\int \frac{1}{x^2+16}\,dx$$

$$= \frac{7}{2}\int \frac{1}{u}\,du - 6\left(\frac{1}{4}\right)\arctan\left(\frac{x}{4}\right) + C$$

$$= \boxed{\frac{7}{2}\ln|x^2+16| - \frac{3}{2}\arctan\left(\frac{x}{4}\right) + C}$$

9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.35.pg

Book Problem 35

Make the substitution $u = \sqrt{x}$ to express the integrand as a rational function and then evaluate the integral $\int \frac{7\sqrt{x}}{x+16}\,dx =$ ______________ +C

Let $u = \sqrt{x} \Rightarrow du = \frac{1}{2}x^{-\frac{1}{2}}dx$

$du = \frac{1}{2\sqrt{x}}dx = \frac{1}{2u}dx$

$\Rightarrow dx = 2u\,du$

$$\int \frac{7\sqrt{x}}{x+16}dx = \int \frac{7u}{u^2+16}\cdot 2u\,du = \int \frac{14u^2}{u^2+16}du = 14\int \frac{u^2}{u^2+16}du$$

$$= 14\int \frac{u^2+16-16}{u^2+16}du$$

$$= 14\int\left(\frac{u^2+16}{u^2+16} - \frac{16}{u^2+16}\right)du$$

$$= 14\int\left(1 - \frac{16}{u^2+16}\right)du$$

$$= 14\left[\int 1\,du - 16\int \frac{1}{u^2+16}du\right]$$

$$= 14\left[u - 16\left(\frac{1}{4}\right)\tan^{-1}\left(\frac{u}{4}\right)\right] + C$$

$$= 14u - \frac{224}{4}\tan^{-1}\left(\frac{u}{4}\right) + C$$

$$= \boxed{14\sqrt{x} - 56\tan^{-1}\left(\frac{\sqrt{x}}{4}\right) + C}$$

10. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.39.pg

Book Problem 39

$$u^2 = (e^x)^2 = e^{2x}$$

Make the substitution $u = e^x$ to express the integrand as a rational function with three linear factors in the denominator, one of
2
which is u, and then evaluate the integral.

$\int \frac{-28e^x - 120}{e^{2x} + 10e^x + 24} dx =$ ______________ +C.

Let $u = e^x$

$du = e^x dx$

$dx = \frac{du}{e^x} = \frac{du}{u}$

$$= \int \frac{-28u - 120}{u^2 + 10u + 24} \cdot \frac{du}{u} = \int \frac{-28u - 120}{(u+6)(u+4)u} du = \int \left(\frac{A}{u+6} + \frac{B}{u+4} + \frac{C}{u} \right) du$$

$$\frac{-28u - 120}{''} = \frac{A(u+4)u + B(u+6)u + C(u+6)(u+4)}{(u+6)(u+4)u}$$

Let $u = -4$: $112 - 120 = 0A - 8B + 0C$

$\Rightarrow \boxed{B = 1}$

11. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.3.Extra2.pg

Extra Problem 2

Evaluate the integral $\int \frac{10}{x^2 + 10x + 50} dx =$ ______________
+C

$$= \int \frac{10}{(x^2 + 10x + 25) - 25 + 50} dx$$

$$= \int \frac{10}{(x+5)^2 + 25} dx$$

Let $u = x + 5$

$du = dx$

$$= \int \frac{10}{u^2 + 25} du = 10 \int \frac{1}{u^2 + 5^2} du$$

$$= 10 \left(\frac{1}{5} \right) \arctan\left(\frac{u}{5} \right) + C$$

$$= \boxed{2 \arctan\left(\frac{x+5}{5} \right) + C}$$