Section 6.5

1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.5.7ab.pg

Book Problem 7

Approximate the integral $\int_1^5 \sqrt{3+x^2}\,dx$ with $n = 4$ using the specified method. (Round your answers to six decimal places.)

1 2 3 4 5

The Right Riemann Sum $R_4 = \Delta x(f(2)+f(3)+f(4)+f(5)) = (1)(\sqrt{7}+\sqrt{12}+\sqrt{19}+\sqrt{28}) = 15.760294$

The Left Riemann Sum $L_4 = \Delta x(f(1)+f(2)+f(3)+f(4)) = (1)(\sqrt{4}+\sqrt{7}+\sqrt{12}+\sqrt{19}) = 12.4687518$

The Midpoint Rule $M_4 = \Delta x(f(1.5)+f(2.5)+f(3.5)+f(4.5)) = (1)(\dots\dots) = 14.0596193$

The Trapezoidal Rule $T_4 = \frac{\Delta x}{2}(f(1)+2f(2)+2f(3)+2f(4)+f(5)) = \frac{1}{2}(\sqrt{4}+2\sqrt{7}+2\sqrt{12}+2\sqrt{19}+\sqrt{28}) = 14.1145$

Simpson's Rule $S_4 = \frac{\Delta x}{3}(f(1)+4f(2)+2f(3)+4f(4)+f(5)) = \frac{1}{3}(\sqrt{4}+4\sqrt{7}+2\sqrt{12}+4\sqrt{19}+\sqrt{28}) =$
$= 14.0794356$

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2. (2 pts) UNCC1242/EssentialCalculus-Stewart-Sec6.5.18ab.pg

Book Problem 18

0 $\frac{1}{5}$ $\frac{2}{5}$ $\frac{3}{5}$ $\frac{4}{5}$ 1

$\Delta x = \frac{b-a}{n} = \frac{1}{5} = .2$

(a) Use the Trapezoidal Rule, with $n = 5$, to approximate the integral $\int_0^1 8\cos(2x)dx$. $f(x) = 8\cos(2x)$
$T_5 = 3.588564028$

$T_5 = \frac{\Delta x}{2}\left[f(0)+2f(\tfrac{1}{5})+2f(\tfrac{2}{5})+2f(\tfrac{3}{5})+2f(\tfrac{4}{5})+f(1)\right]$
$= \frac{.2}{2}\left[8\cos 0+16\cos(\tfrac{2}{5})+16\cos(\tfrac{4}{5})+16\cos(\tfrac{6}{5})+16\cos(\tfrac{8}{5})+8\cos(2)\right] = \boxed{3.588564028}$

(b) The actual value of $\int_0^1 8\cos(2x)dx = $ 3.63751897
$= 4\sin(2x)\big|_0^1 = 4\sin 2$

(c) The error involved in the approximation of part (a) is $E_T = \int_0^1 8\cos(2x)dx - T_5 =$ ______.

$= 3.63751897 - 3.588564028$
$= \boxed{.0489549 4}$ actual error

(d) The second derivative $f''(x) = \boxed{-32\cos(2x)}$
The value of $K = \max|f''(x)|$ on the interval [0, 1] = $\boxed{32}$

$f(x) = 8\cos(2x)$
$f'(x) = -16\sin(2x)$
Sketch the graph of $Y = |-32\cos(2x)|$ to find K

(e) Find a sharp upper bound for the error in the approximation of part (a) using the Error Bound Formula $|E_T| \le \frac{K(b-a)^3}{12n^2} =$ $\boxed{0.10666\dots}$

$\frac{32(1-0)^3}{12(5)^2} = \frac{32}{12(25)} = 0.106666\dots$

(f) Find the smallest number of partitions n so that the approximation T_n to the integral is guaranteed to be accurate to within 0.001. $n =$ 52

$|E_T| \le 0.001$

$\frac{32(1-0)^3}{12n^2} \le 0.001 \Rightarrow 32 \le 0.012n^2$

$\Rightarrow n^2 \ge \frac{32}{0.012} = 2666.667$

$\Rightarrow n \ge \boxed{51.6397}$

3. (2 pts) UNCC1242/EssentialCalculus-Stewart-Sec6.5.19ab.pg

Book Problem 19

(a) Use Simpson's Rule, with $n = 6$, to approximate the integral $\int_0^1 8e^{-3x}dx$.

$S_6 = \boxed{2.53475346}$ $f(x) = 8e^{-3x}$

$\Delta x = \frac{b-a}{6} = \frac{1}{6}$

$0 \quad \frac{1}{6} \quad \frac{1}{3} \quad \frac{1}{2} \quad \frac{2}{3} \quad \frac{5}{6} \quad 1$

$S_6 = \frac{\Delta x}{3}\left[f(0) + 4f(\tfrac{1}{6}) + 2f(\tfrac{1}{3}) + 4f(\tfrac{1}{2}) + 2f(\tfrac{2}{3}) + 4f(\tfrac{5}{6}) + f(1)\right]$

$= \frac{1/6}{3}\left[8e^0 + 32e^{-3/6} + 16e^{-3/3} + 32e^{-3/2} + 16e^{-6/3} + 32e^{-15/6} + 8e^{-3}\right] = \boxed{2.53475546}$

(b) The actual value of $\int_0^1 8e^{-3x}dx = \frac{8e^{-3x}}{-3}\Big|_0^1 = \frac{8e^{-3}}{-3} - \frac{8}{-3} = \boxed{2.53390115}$

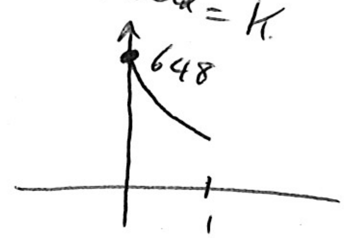
(c) The error involved in the approximation of part (a) is $E_S = \int_0^1 8e^{-3x}dx - S_6 = -0.0008543$

(d) The fourth derivative $f^{(4)}(x) = 648e^{-3x}$.

The value of $K = \max |f^{(4)}(x)|$ on the interval $[0, 1]$ = 648.

$f(x) = 8e^{-3x} \to f'(x) = -24e^{-3x} \to f''(x) = 72e^{-3x} \to f'''(x) = -216e^{-3x} \to f^{(4)}(x) = 648e^{-3x}$

to find K, graph $Y = |648e^{-3x}|$ on the interval $[0, 1]$, find the max $= K$.



(e) Find a sharp upper bound for the error in the approximation of part (a) using the Error Bound Formula $|E_S| \le \frac{K(b-a)^5}{180n^4} =$

$|E_S| \le \frac{648(1-0)^5}{180(6)^4} = 0.0027777\ldots$

(f) Find the smallest number of partitions n so that the approximation S_n to the integral is guaranteed to be accurate to within 0.0001. $n =$ 14 (should always be even)

$|E_S| \le 0.0001$

$\frac{648(1-0)^5}{180n^4} \le 0.0001 \Rightarrow n^4 \ge \frac{648}{180(0.0001)} = 36000 \Rightarrow n \ge 13.77$

$\boxed{n = 14}$

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.5.20ab.pg

Book Problem 20

Find the smallest number of partitions n so that the approximation to $\int_1^8 2\ln x\,dx$ using Simpson's Rule is accurate to within 0.0001?

Answer: $n =$ ______.

$|E_S| \le \frac{K(b-a)^5}{180n^4}$, $f(x) = 2\ln x \Rightarrow f'(x) = \frac{2}{x} = 2x^{-1}$

$f''(x) = \frac{-2}{x^2} = -2x^{-2}$

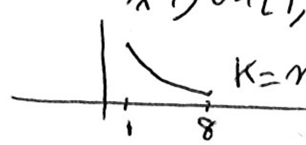
$f'''(x) = 4x^{-3} \Rightarrow f^{(4)}(x) = -12x^{-4}$

$f^{(4)}(x) = \frac{-12}{x^4}$

$|E_S| \le 0.0001$

$\frac{12(8-1)^5}{180n^4} \le 0.0001 \Rightarrow \frac{12(7)^5}{180(0.0001)} \le n^4$

graph $Y_1 = \text{abs}\left(\frac{-12}{x^4}\right)$ on $[1, 8]$



$K = \max = 12$

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.5.29ab.pg

Book Problem 29

Use Simpson's Rule and all the data in the following table to estimate the value of the integral $\int_{-1}^{5} y\,dx$. Here $y = f(x)$ and with values given in the table.

x	-1	0	1	2	3	4	5
y	-8	-5	-1	0	4	7	9

$S_6 = \frac{\Delta x}{3}\left[f(-1) + 4f(0) + 2f(1) + \cdots\right]$

$= \frac{1}{3}\left[-8 + 4(-5) + 2(-1) + 4(0) + 2(4) + 4(7) + 9\right]$

$= \frac{1}{3}\left[-8 - 20 - 2 + 8 + 28 + 9\right] = 5$

The Simpson's Rule approximation = 5

6. (2 pts) UNCC1242/EssentialCalculus-Stewart-Sec6.5.17ab.pg

0.5 1.5 2.5 3.5
0 4

Book Problem 17

(a) Use the Midpoint Rule, with $n = 4$, to approximate the integral $\int_0^4 7e^{-x^2}dx$.
$M_4 =$ 6.202946728 (Round your answers to six decimal places.)

$M_4 = \Delta x[f(0.5)+f(1.5)+f(2.5)+f(3.5)]$
$= 1[7e^{-(0.5)^2} + 7e^{-(1.5)^2} + \cdots] = \boxed{6.202946728..}$

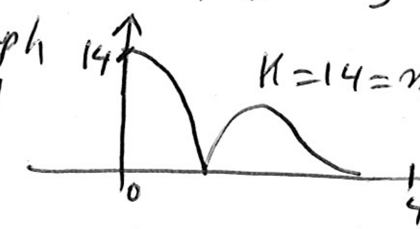
(b) Compute the value of the definite integral in part (a) using your calculator, such as MATH 9 on the TI83/84 or 2ND 7 on the TI-89. $\int_0^4 7e^{-x^2}dx =$ 6.2035 88383

Math 9 → fnInt(7e^(−X2), X, 0, 4) $\boxed{\text{enter}}$

(c) The error involved in the approximation of part (a) is $E_M = \int_0^4 7e^{-x^2}dx - M_4 =$ 6.2035 8838 − 6.20294673 = ...

$f(x) = 7e^{-x^2}$
$f'(x) = -14xe^{-x^2}$
$f''(x) = -14e^{-x^2} + 28x^2e^{-x^2}$
Y = abs(− // + //)
graph on [0, 4]

(d) The second derivative $f''(x) =$ $-14e^{-x^2} + 28x^2e^{-x^2}$.
The value of $K = \max|f''(x)|$ on the interval $[0, 4] =$ 14.



K = 14 = max

(e) Find a sharp upper bound for the error in the approximation of part (a) using the Error Bound Formula $|E_M| \le \frac{K(b-a)^3}{24n^2}$ $= \frac{14(4-0)^3}{24(4)^2}$
2.33333 (where a and b are the lower and upper limits of integration, n the number of partitions used in part a).

(f) Find the smallest number of partitions n so that the approximation M_n to the integral is guaranteed to be accurate to within 0.001. $n =$ 194

$|E_M| \le 0.001$
$\frac{14(4-0)^3}{24n^2} \le 0.001 \Rightarrow \frac{14(64)}{24(0.001)} \le n^2$
$\Rightarrow n^2 \ge 37333.333$
$\Rightarrow n \ge 193.218 \Rightarrow n = 194$

7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.5.Extraab.pg

Extra Problem

A student is speeding down Highway 16 in her fancy red Porsche when her radar system warns her of an obstacle 400 feet ahead. She immediately applies the brakes, starts to slow down, and spots a skunk in the road directly ahead of her. The "black box" in the Porsche records the car's speed every two seconds, producing the following table. The speed decreases throughout the 10 seconds it takes to stop, although not necessarily at a uniform rate.

Time since brakes applied (sec)	0	2	4	6	8	10
Speed (ft/sec)	105	85	50	25	5	0

A. What is your best estimate of the total distance the student's car traveled before coming to rest (note that the best estimate is probably not the over or under estimate that you can most easily find, use the trapezoidal approximation)?
distance = $T_5 = \frac{\Delta x}{2}[f(0)+2f(2)+2f(4)+2f(6)+2f(8)+f(10)] = \frac{2}{2}[105+2(85)+2(50)+2(25)+2(5)+0]$
$= 435$

B. Given the fact that the Porsche slows down during breaking, give a sharp

i. *underestimate* of the distance traveled: $R_5 = (85+50+25+5+0)(2) = 330$

ii. *overestimate* of the distance traveled: $L_5 = (105+85+50+25+5)(2) = 540$

C. Which one of the following statements can you justify from the information given?

- A. The "black box" data is inconclusive. The skunk may or may not have been hit.
- B. The car stopped before getting to the skunk.
- C. The skunk was hit by the car.