WeBWorK assignment number Sec6.6 is due : 10/02/2010 at 11:30pm EDT.

There are two types of **Improper Integrals** :

Type 1: Infinite Intervals

(a) $\int_a^{\infty} f(x)dx = \lim_{t\to\infty} \int_a^t f(x)dx$

(b) $\int_{-\infty}^{b} f(x)dx = \lim_{t\to-\infty} \int_t^b f(x)dx$

These improper integrals are called **convergent** if the corresponding limit exists and **divergent** if the limit does not exist.

(c) $\int_{-\infty}^{\infty} f(x)dx = \int_{-\infty}^{a} f(x)dx + \int_a^{\infty} f(x)dx$ if a is any real number and both integrals are convergent.

Type 2: Discontinuous Integrands

(a) If f is continuous on $[a,b)$ and discontinuous at b, then $\int_a^b f(x)dx = \lim_{t\to b^-} \int_a^t f(x)dx$

(b) If f is continuous on $(a,b]$ and discontinuous at a, then $\int_a^b f(x)dx = \lim_{t\to a^+} \int_a^t f(x)dx$

(c) If f is discontinuous at c, where $a<c<b$, then $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$

Comparison Theorem Suppose that f and g are continuous and $f(x) \geq g(x) \geq 0$ for $x \geq a$, then

(a) If $\int_a^{\infty} f(x)dx$ is convergent, then $\int_a^{\infty} g(x)dx$ is convegrent.

(b) If $\int_a^{\infty} g(x)dx$ is divergent, then $\int_a^{\infty} f(x)dx$ is divegrent.

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1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.3.pg

Book Problem 3

A) The area under the curve $y = \frac{9}{x^2}$ from $x = 1$ to $x = t$ is equal to ______.

$= \int_1^t \frac{9}{x^2}dx = \frac{-9}{x}\Big|_1^t = \boxed{\frac{-9}{t} + 9}$

B) The area under the curve from $x = 1$ to $x = 10$ is equal to ______.

$= \frac{-9}{10} + 9 = \frac{-9+90}{10} = \boxed{8.1}$

C) The area under the curve from $x = 1$ to $x = 100$ is equal to ______.

$= \frac{-9}{100} + 9 = \frac{-9+900}{100} = \boxed{8.91}$

D) The area under the curve from $x = 1$ to $x = 1000$ is equal to ______.

$= \frac{-9+9000}{1000} = \boxed{8.991}$

E) The total area under this curve for $x \geq 1$ is equal to ______ = lim when $t \to \infty$ = $\boxed{9}$

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2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.5.pg

Book Problem 5

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

let $u = 7x-3 \Rightarrow du = 7dx$

$\int \frac{1}{(7x-3)^4}dx = \frac{-1}{21(7x-3)^3} + C$.

$= \int \frac{1}{u^4} \cdot \frac{du}{7} = \frac{1}{7}\int u^{-4}du = \frac{1}{7}\left(\frac{u^{-3}}{-3}\right) + C = \frac{-1}{21(7x-3)^3} + C$

$\int_1^{\infty} \frac{1}{(7x-3)^4}dx =$ ~~$\frac{1}{1344}$~~ .

$= \lim_{t\to\infty} \int_1^t \frac{1}{(7x-3)^4}dx = \lim_{t\to\infty} \left(\frac{-1}{21(7x-3)^3}\right)\Big|_1^t = \lim_{t\to\infty}\left(\frac{-1}{21(7t-3)^3} - \frac{-1}{21(4)^3}\right) = \boxed{0 + \frac{1}{21(64)}}$

16. **(1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.43.pg**

Book Problems 43

Enter T if the given statement is true or F if it false.
Only three attempts are allowed on this problem.

$\int_1^t \frac{dx}{x}\,dx = \lim_{t\to\infty} \ln|x|\Big|_1^t = \lim_{t\to\infty} \ln t - \ln 1$ ($\ln 1 = 0$)

$= \ln\infty = \infty$

so $\int_1^\infty \frac{dx}{x}$ is DIV

F 1. $\int_1^\infty \frac{dx}{x+e^{7x}}$ is convergent because $\frac{1}{x+e^{7x}} \le \frac{1}{x}$ and $\int_1^\infty \frac{dx}{x}$ is $= \lim_{t\to\infty}$ ~~convergent~~ incorrect

T 2. $\int_1^\infty \frac{dx}{x-e^{7x}}$ is divergent because $\frac{1}{x-e^{7x}} \ge \frac{1}{x}$ and $\int_1^\infty \frac{dx}{x}$ is divergent → so $\int_1^\infty \frac{1}{x-e^{7x}}\,dx \ge \int_1^\infty \frac{1}{x}\,dx \to$ DIV

T 3. $\int_1^\infty \frac{dx}{x+e^{7x}}$ is convergent because $\frac{1}{x+e^{7x}} \le \frac{1}{e^{7x}}$ and $\int_1^\infty \frac{dx}{e^{7x}}$ is convergent $= \lim_{t\to\infty} \int_1^t e^{-7x}\,dx = \lim_{t\to\infty}\left(\frac{e^{-7t}}{-7} - \frac{e^{-7}}{-7}\right) = \frac{e^{-\infty}}{-7} + \frac{e^{-7}}{7}$

$= 0 + \frac{e^{-7}}{7}$

Conv.

F 4. $\int_1^\infty \frac{dx}{x+e^{7x}}$ is divergent because $\frac{1}{x+e^{7x}} \le \frac{1}{x}$ and $\int_1^\infty \frac{dx}{x}$ is divergent

$\int_1^\infty \frac{1}{x+e^{7x}}\,dx \le \int_1^\infty \frac{1}{x}\,dx$ DIV ⇒ No conclusion about the first integral

17. **(1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.55.pg**

Book Problem 55

The mass at time t of a radioactive substance is $m(t) = m(0)e^{kt}$, where $m(0)$ is the initial mass and $k = -0.000139$.

The mean life M of an atom in this substance is $M = -k\int_0^\infty te^{kt}\,dt =$ ____

3. **(1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.6.pg**

$u = 2x-4 \Rightarrow du = 2dx$

Book Problem 6

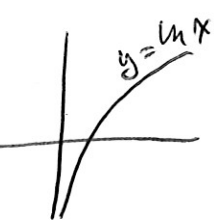
$\int \frac{-2}{2x-4}\,dx = \int \frac{-du}{u} = -\ln|u| + C = -\ln|2x-4| + C$

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges, state your answer as "DIV". $\boxed{\text{DIV}}$

$\int_{-\infty}^{0} \frac{-2}{2x-4}\,dx = \lim_{t\to-\infty} \int_t^0 \frac{-2}{2x-4}\,dx = \lim_{t\to-\infty}\left(-\ln|2x-4|\right)\Big|_t^0 = \lim_{t\to-\infty}\left(-\ln|-4| + \ln|2t-4|\right)$

$= -\ln 4 + \ln\infty$

$= \boxed{\infty}$



4. **(1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.7.pg**

Book Problem 7

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges , state your answer as "DIV".

$\int_{-\infty}^{-3} \frac{6}{\sqrt{5-x}}\,dx =$ ____

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.9.pg

Book Problem 9

$$\int_5^t 6e^{-y/4}\,dy = 6(-4)e^{-y/4}\Big|_5^t = -24e^{-t/4}+24e^{-5/4}$$

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$$\int_5^{\infty} 6e^{-y/4}\,dy = \lim_{t\to\infty}\left(-24e^{-t/4}+24e^{-5/4}\right) = \underbrace{-24e^{-\infty}}_{=0}+24e^{-5/4} = \boxed{24e^{-5/4}}$$

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6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.11.pg

Book Problem 11

$$\int_{2\pi}^t \cos(-8\theta)\,d\theta = \frac{\sin(-8\theta)}{-8}\Big|_{2\pi}^t = \frac{\sin(-8t)}{-8} - \underbrace{\frac{\sin(-16\pi)}{-8}}_{=0}$$

$\sin(\ldots\pi)=0$

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$$\int_{2\pi}^{\infty} \cos(-8\theta)\,d\theta = \lim_{t\to\infty}\left(\frac{\sin(-8t)}{-8}\right) = \text{Does not exist} = \boxed{\text{DIV}}$$

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7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.13.pg

Book Problem 13

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$\int 7xe^{-6x^2}\,dx =$ ________________ +C.

$\int_{-\infty}^{\infty} 7xe^{-6x^2}\,dx =$ ________________.

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8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.17.pg

Book Problem 17

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$\int \frac{\ln(x)}{x}\,dx =$ ________________ +C.

$\int_2^{\infty} \frac{\ln(x)}{x}\,dx =$ ________________.

9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.19.pg

Book Problem 19

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$\int \frac{\ln x}{x^6}\,dx =$ ______________ $+C$.

$\int_1^{\infty} \frac{\ln x}{x^6}\,dx =$ ______________.

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10. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.23.pg

Book Problem 23

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$\int_0^1 \frac{10}{x^2}\,dx = \lim_{t\to 0^+}\int_t^1 10x^{-2}\,dx = \lim_{t\to 0^+} \frac{10x^{-1}}{-1}\Big|_t^1 = \lim_{t\to 0^+} \frac{-10}{x}\Big|_t^1 = \lim_{t\to 0^+}\left(\frac{-10}{1}+\frac{10}{t}\right) = \boxed{\text{DIV}}$

$+\infty$

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11. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.25.pg

Book Problem 25

Determine whether the integral is divergent or convergent. If it is convergent, evaluate it. If it diverges, state your answer as "DIV".

$\int_4^{17} \frac{11}{\sqrt[8]{x-4}}\,dx =$ ______________

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12. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.28.pg

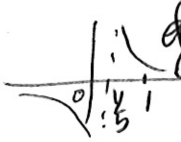
Book Problem 28

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$\int \frac{3}{5y-1}\,dy = \frac{3\ln|5y-1|}{5} + C$.

$\int_0^1 \frac{3}{5y-1}\,dy = \int_0^{1/5} + \int_{1/5}^1 = \text{DIV}$

∴ function is discontinuous at $\frac{1}{5}$



$\int_0^{\frac{1}{5}} \frac{3}{5y-1}\,dy = \lim_{t\to\frac{1}{5}} \int_0^t \frac{3}{5y-1}\,dy$

$= \lim_{t\to\frac{1}{5}} \frac{3\ln|5y-1|}{5}\Big|_0^t = \cancel{\lim_{t\to\frac{1}{5}} \frac{3\ln|5t-1|}{5} - 3}$

$= \lim_{t\to\frac{1}{5}} \left(\frac{3\ln|5t-1|}{5} - \overbrace{\frac{3\ln 1}{5}}^{0}\right)$

$= \frac{3\ln 0}{5} - 0$

$= -\infty$

$\ln 0 = -\infty$

13. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.29.pg

Book Problem 29

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If not, state your answer as "DIV".

$\int \frac{13e^x}{e^x - 1} dx =$ ________________+C.

$\int_{-3}^{3} \frac{13e^x}{e^x - 1} dx =$ ________________.

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14. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.31.pg

Book Problem 31

Evaluate the indefinite integral and determine whether the corresponding improper integral is divergent or convergent. If it is convergent, evaluate it. If it diverges, state your answer as "DIV".

$\int z^7 \ln z dz =$ ________________+C.

$\int_0^9 z^7 \ln z dz =$ ________________.

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15. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec6.6.41.pg

Book Problems 41

Enter T if the given statement is true or F if it false.
Only three attempts are allowed on this problem.

___1. $\int_1^\infty \frac{\sin^2 x}{1+x^2}\, dx$ is convergent because $\frac{\sin^2 x}{1+x^2} \leq \frac{1}{1+x^2}$ and $\int_1^\infty \frac{1}{1+x^2}\, dx$ is convergent.

___2. $\int_1^\infty \frac{\sin^2 x}{1+x^2}\, dx$ is convergent because $\frac{\sin^2 x}{1+x^2} \leq \frac{1}{x^2}$ and $\int_1^\infty \frac{1}{x^2}\, dx$ is convergent.

___3. $\int_1^\infty \frac{\sin^2 x}{1+x^2}\, dx$ is convergent because $\frac{\sin^2 x}{1+x^2} \leq \sin^2 x$ and $\int_1^\infty \sin^2 x\, dx$ is convergent.

___4. $\int_1^\infty \frac{\sin^2 x}{1+x^2}\, dx$ is divergent because $\frac{\sin^2 x}{1+x^2} \geq \frac{1}{1+x^2}$ and $\int_1^\infty \frac{1}{1+x^2}\, dx$ is divergent.