

Book Problem 15

Consider the region enclosed by the graphs of $2y = 3\sqrt{x}$, $y = 8$, and $2y + 1x = 4$.

The area A of this region could be computed in two ways:

1) By integrating with respect to x , $A = \int_a^b h_1(x) dx + \int_b^c h_2(x) dx$

where $a = -12$, $b = 1$, $c = 28.4444...$

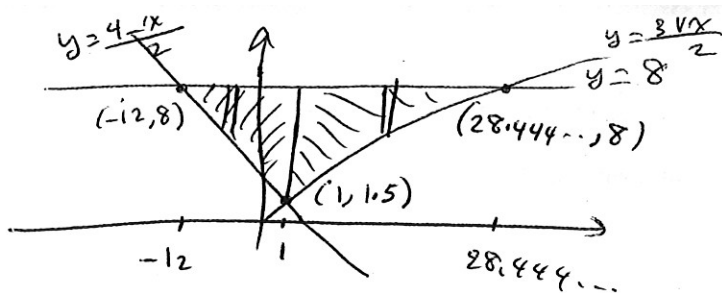
$h_1(x) = 8 - (4 - \frac{x}{2})$, and $h_2(x) = 8 - 3\sqrt{\frac{x}{2}}$

The area $A =$ _____

2) By integrating with respect to y , $A = \int_d^e h_3(y) dy$

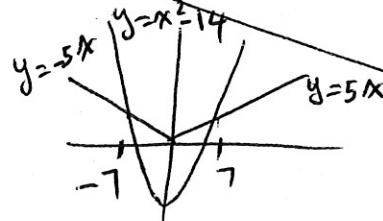
where $d = 1.5$, $e = 8$, and $h_3(y) = \frac{4y^2}{9} - 4 + 2y$

The area $A =$ _____



$$\begin{aligned} 1) A &= \int_{-12}^1 (8 - (4 - \frac{x}{2})) dx + \int_1^{28.444...} (8 - 3\sqrt{\frac{x}{2}}) dx \\ &= \int_{-12}^1 (6 + \frac{x}{2}) dx + \dots \\ &= (6x + \frac{x^2}{4}) \Big|_{-12}^1 + (8x - \frac{3}{2} \cdot \frac{2}{3} x^{3/2}) \Big|_1^{28.444...} \end{aligned}$$

$$\begin{aligned} 2) A &= \int_{1.5}^8 (x_{\text{right}} - x_{\text{left}}) dy \\ &= \int_{1.5}^8 (\frac{4y^2}{9} - (4 - 2y)) dy \\ &= \int_{1.5}^8 (\frac{4y^2}{9} - 4 + 2y) dy \\ &= (\frac{4y^3}{27} - 4y + y^2) \Big|_{1.5}^8 \end{aligned}$$



Sketch the region enclosed by the curves $y = 5|x|$ and $y = x^2 - 14$.

List the points of intersection of these curves from LEFT to RIGHT in the form (x, y) :

$(-7, 35)$ and $(7, 35)$.

Due to the symmetry of the region with respect to the y -axis, the area

$A = 2 \int_0^a h(x) dx$ where $a = 7$ and $h(x) = 5x - (x^2 - 14)$

After integrating, the area $A =$ _____

$$\begin{aligned} A &= \int_0^7 (y_{\text{top}} - y_{\text{bottom}}) dx = \int_0^7 (5x - x^2 + 14) dx \\ &= (\frac{5x^2}{2} - \frac{x^3}{3} + 14x) \Big|_0^7 = \dots \end{aligned}$$

Book Problem 21

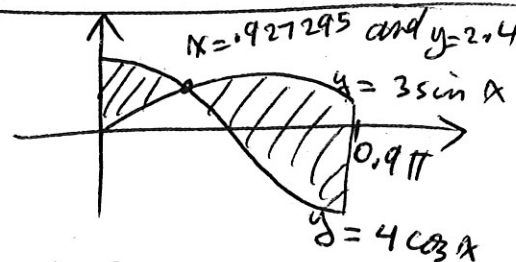
Sketch the region that lies between the curves $y = 3\sin(x)$ and $y = 4\cos(x)$ from $x = 0$ to $x = 0.9\pi$. Notice that this region

consists of two separate parts.

Using a graphing calculator, the x -coordinate c of the point of intersection is approximately equal to 0.927295

The area A of this region is $= \int_0^c \text{_____} dx + \int_c^{0.9\pi} \text{_____} dx$

After integrating, $A =$ _____



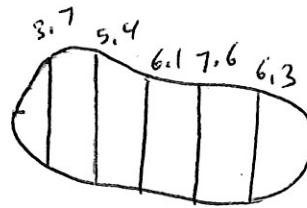
$$\begin{aligned} A &= \int_0^{0.927295} (4\cos x - 3\sin x) dx \\ &+ \int_{0.927295}^{0.9\pi} (3\sin x - 4\cos x) dx \end{aligned}$$

Book Problem 25

The widths (in meters) of a kidney-shaped swimming pool, measured at 2-meter intervals, are:

0, 3.7, 5.4, 6.1, 7.6, 6.3, and 0.

Use Simpson's Rule to estimate the area of the pool.



$$\begin{aligned} \text{Area} &= \int [f(x) - g(x)] dx \\ &= \frac{\Delta x}{3} [0 + 4(3.7) + 2(5.4) + 4(6.1) + 2(7.6) + 4(6.3) + 0] \\ &= \frac{2}{3} (14.8 + 10.8 + 24.4 + 15.2 + 25.2) \\ &= \boxed{\dots} \end{aligned}$$

10. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.1.27.pg

Book Problem 27

If the birth rate of a population is $b(t) = 3000e^{0.029t}$ people per year and the death rate is $d(t) = 1530e^{0.013t}$ people per year, then the increase in population over a 5-year period is equal to the area between these curves for $0 \leq t \leq 5$.

$$\begin{aligned} &\int_0^5 (3000e^{0.029t} - 1530e^{0.013t}) dt \quad \text{integrate} \\ &= \frac{3000}{0.029} (e^{0.029 \times 5} - e^0) - \frac{1530}{0.013} (e^{0.013 \times 5} - e^0) \\ &= \dots \end{aligned}$$

Thus, the increase in population during that period is equal to $\int_0^5 \dots dt = \dots$

11. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.1.31.pg

Book Problem 31

Find the value of $c > 0$ such that the area of the region enclosed by the parabolas $y = x^2 - c^2$ and $y = c^2 - x^2$ is 140.

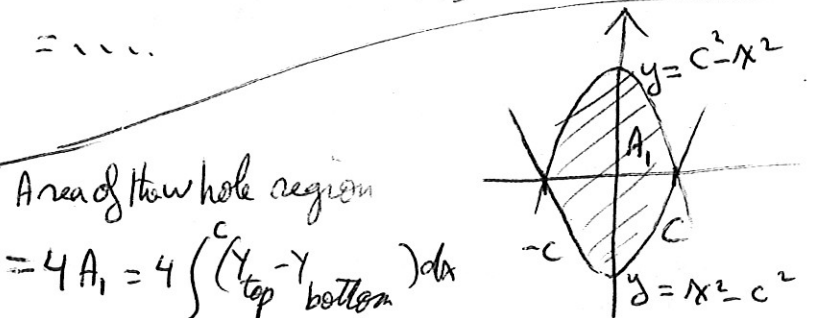
$c = \dots$

12. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.1.33.pg

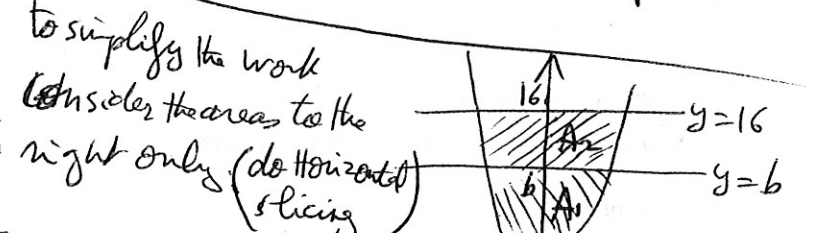
Book Problem 33

Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 16$ into two regions with equal area.

$b = \dots$



$$\begin{aligned} &\text{Area of the whole region} \\ &= 4A_1 = 4 \int_0^c (y_{\text{top}} - y_{\text{bottom}}) dx \\ &= 4 \int_0^c (c^2 - x^2 - 0) dx = 4 \left(c^2x - \frac{x^3}{3} \right) \Big|_0^c = 4 \left(c^3 - \frac{c^3}{3} \right) \\ &= \frac{8}{3} c^3 = 140 \Rightarrow c^3 = \frac{3(140)}{8} \Rightarrow c = \sqrt[3]{\dots} \end{aligned}$$



$$\begin{aligned} &\text{to simplify the work} \\ &\text{consider the area to the right only (do horizontal slicing)} \\ &A_1 = A_2 \\ &A_1 = \int_0^b (x_{\text{right}} - x_{\text{left}}) dy = \int_b^{16} (x_{\text{right}} - x_{\text{left}}) dy = A_2 \\ &\int_0^b (\sqrt{y} - 0) dy = \int_b^{16} (\sqrt{y} - 0) dy \\ &\Rightarrow \frac{2}{3} y^{3/2} \Big|_0^b = \frac{2}{3} y^{3/2} \Big|_b^{16} \\ &\Rightarrow \frac{2}{3} b^{3/2} = \frac{2}{3} (16^{3/2} - b^{3/2}) = \frac{2}{3} (64 - b^{3/2}) \\ &\Rightarrow b^{3/2} = 64 - b^{3/2} \Rightarrow 2b^{3/2} = 64 \Rightarrow b^{3/2} = 32 \Rightarrow b = (32)^{2/3} \end{aligned}$$