Wafaa Shaban

WeBWorK assignment number Sec7.2 is due : 10/19/2010 at 11:30pm EDT.

The Volume V of a solid obtained by rotating a region about an axis:

Case a) If the rotation is about the x-axis or a line parallel to the x-axis, take vertical cross-sections, and $V = \int_a^b A(x)dx$

Case b) If the rotation is about the y-axis or a line parallel to the y-axis, take horizontal cross-sections, and $V = \int_a^b A(y)dy$

where $A = \pi(radius)^2$ if the cross-section is a disk,
and $A = \pi(outer\ radius)^2 - \pi(inner\ radius)^2$ if the cross section is a washer.

In case a) the *radius* should be in terms of x. (Solve for y as a function of x).
In case b) the *radius* should be in terms of y. (Solve for x as a function of y).

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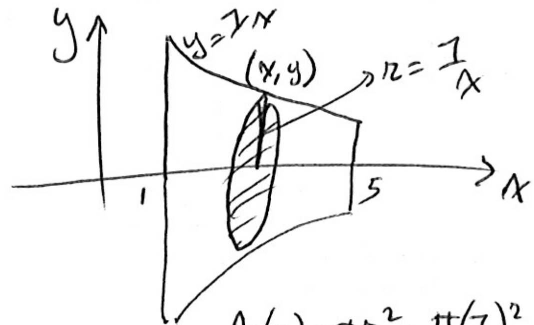
Book Problem 1



Find the volume V of the solid obtained by rotating the region bounded by the curves

$y = 7/x, x = 1, x = 5, y = 0$; about the x-axis.

$V=\int_1^5 A(x)dx$

$A(x)=\pi r^2=\pi\left(\frac{7}{x}\right)^2$

The volume V = $\int_1^5 \frac{49\pi}{x^2}\,dx$.

$=\int_1^5 \pi\left(\frac{49}{x^2}\right)dx=\frac{-49\pi}{x}\Big|_1^5=\frac{-49\pi}{5}+\frac{49\pi}{1}$

Therefore V= $49\pi\left(\frac{4}{5}\right)$

$=49\pi\left(1-\frac{1}{5}\right)=49\pi\left(\frac{4}{5}\right)$

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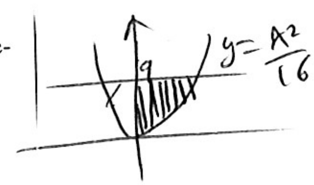
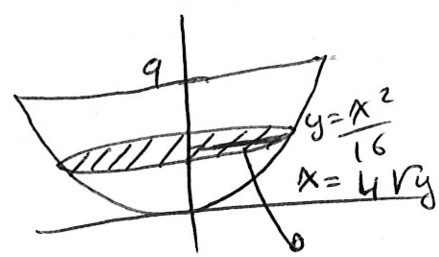
2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.3.pg

Book Problem 3





Find the volume V of the solid obtained by rotating the region bounded by the curves

$x = 4\sqrt{y}, x = 0, y = 9$; about the y-axis.

$y=\frac{x^2}{16}$

V = $\int_a^b h(y)dy$ where $a = \underline{0}$, $b = \underline{9}$, $h(y) = \underline{16\pi y}$

V= $8(81)\pi$

$V=\int_0^9 A(y)dy$

$r=4\sqrt{y}$

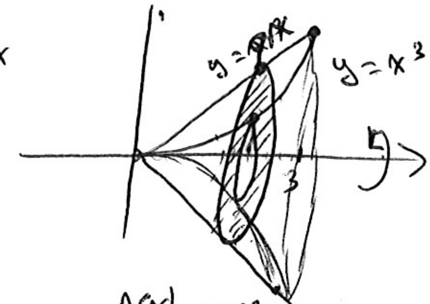
$=\int_0^9 16\pi y\,dy$

$A(y)=\pi r^2=16\pi y$

$=8\pi y^2\Big|_0^9=8\pi(81)$

$x^3 = 9x$

$x^2 = 9 \Rightarrow x = \pm 3$



Washer

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.5.pg

Book Problem 5

Find the volume V of the solid formed by rotating the region inside the first quadrant enclosed by $y = x^3$ and $y = 9x$; about the x-axis.

$V = \int A(x)\,dx$

$A(x) = \pi R^2 - \pi r^2$

$V = \int_0^3 \pi(81x^2 - x^6)\,dx$

$= \pi(9x)^2 - \pi(x^3)^2$

$V = \int_a^b h(x)dx$ where $a =$ 0, $b =$ 3, $h(x) =$ $\pi(81x^2 - x^6)$

$= \pi\left(\frac{81x^3}{3} - \frac{x^7}{7}\right)\Big|_0^3 = \ldots$

V= ...

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.7.pg

Book Problem 7

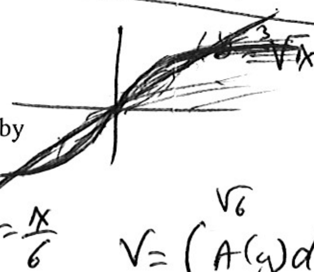
Find the volume V formed by rotating the region enclosed by the curves:

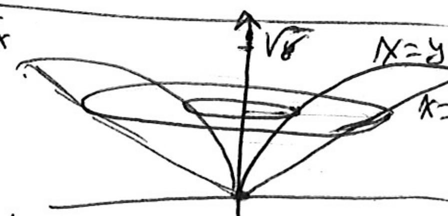
$\rightarrow x = y^3$

$y^3 = x$ and $x = 6y$ with $y \geq 0$; about the y-axis.

$y^3 = 6y \Rightarrow y = \pm\sqrt{6}$

V = $y^2 = 6$





$V = \int_0^{\sqrt{6}} A(y)\,dy$

$A(y) = \pi R^2 - \pi r^2$

$V = \int_0^{\sqrt{6}} \pi(36y^2 - y^6)\,dy$

$= \pi(6y)^2 - \pi(y^3)^2$

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.9.pg

Book Problem 9

Find the volume of the solid obtained by rotating the region bounded by the curves:

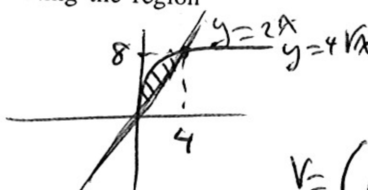
$2x = 4\sqrt{x}$

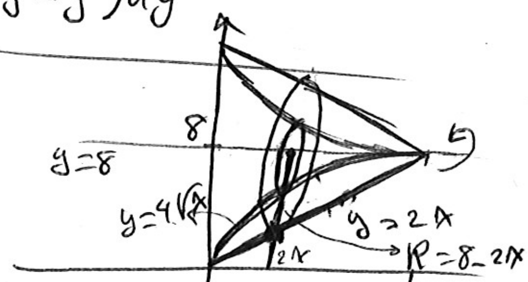
$4x^2 = 16x$

$y = 2x$, $y = 4\sqrt{x}$; about $y = 8$.

$x = 4$

Volume = ____________





$V = \int_0^4 A(x)\,dx$

$A(x) = \pi R^2 - \pi r^2$

$V = \pi \int_0^4 \left[(8-2x)^2 - (8-4\sqrt{x})^2\right] dx$

$= \pi(8-2x)^2 - \pi(8-4\sqrt{x})^2$

$= \pi \int_0^4 (\cancel{64} - 32x + 4x^2 \cancel{-64} + 64\sqrt{x} - 16x)\,dx$

$= \pi\left(-\frac{32x^2}{2} + \frac{4x^3}{3} - \ldots\right)\Big|_0^4$

6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.10.pg

Book Problem 10

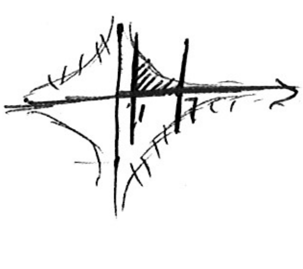
Find the volume of the solid obtained by rotating the region bounded by the curves:

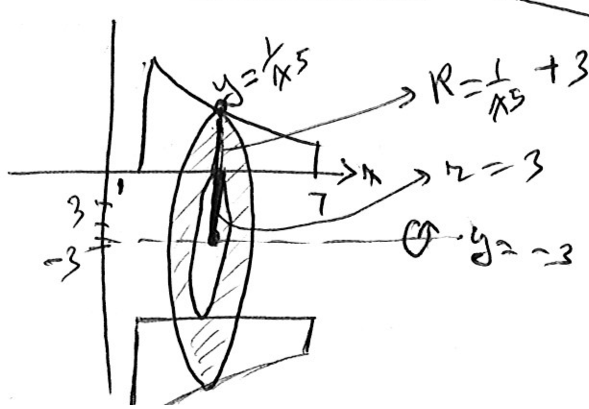
x-axis

$y = 1/x^5$, $y = 0$, $x = 1$, $x = 7$; about $y = -3$.

Volume = $\int_1^7$ ____________ dx.

Thus the volume = ____________.





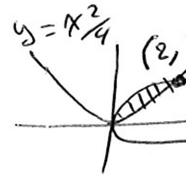
$\rightarrow R = \frac{1}{x^5} + 3$

$\rightarrow r = 3$

$y = -3$

$A(x) = \pi\left(\frac{1}{x^5} + 3\right)^2 - \pi(3)^2$

7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.11.pg



intersection

$x = 2y^2 = 2\left(\frac{x^2}{4}\right)^2 = \frac{x^4}{8}$

$8x = x^4$

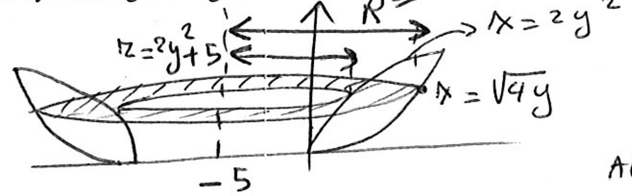
$8 = x^3 \Rightarrow x = 2$

Book Problem 11

Find the volume of the solid obtained by rotating the region bounded by the curves:

$y = x^2/4, x = 2y^2$; about $x = -5$.

$x = \sqrt{4y}$, $x = 2y^2$



$A(y) = \pi R^2 - \pi r^2$

$A(y) = \pi\left(\sqrt{4y}+5\right)^2 - \pi\left(2y^2+5\right)^2$

$A(y) = \pi\left[4y + 20\sqrt{y} + 25 - 4y^4 - 20y^2 - 25\right]$

$= \pi[\ldots\ldots]$

$V = \int_0^1 A(y)\,dy$

Volume: ______

8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.23.pg

Book Problem 23

Suppose that a CAT scan of a human liver shows cross-sections spaced 1.6 cm apart. The liver is 12.8 cm long and the cross-sectional areas, in square centimeters are 0, 12, 51, 75, 94, 101, 68, 33, and 0. Use Simpson's Rule to estimate the volume of the liver.

$V = \int_0^{12.8} A(x)\,dx \approx S_{\ldots}$

$= \frac{\Delta x}{3}\left[A(x_0) + 4A(x_1) + 2A(x_2) + 4\ldots + 4\ldots + A(x_n)\right]$

$= \frac{1.6}{3}\left[0 + 4(12) + 2(51) + 4(75) + 2(94) + 4(101) + 2(68) + 4(33) + 0\right]$

$= \ldots\ldots$

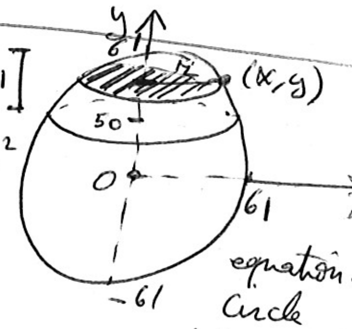
Answer: ______

9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.2.27.pg

Book Problem 27

Find the volume of a cap of a sphere with radius $r = 61$ and height $h = 11$.

$V = \int_{50}^{61} A(y)\,dy$ $\quad h = 11$



$A(y) = \pi r^2 = \pi\left(\sqrt{61^2 - y^2}\right)^2$

$= \pi(61^2 - y^2)$

$V = \int_{50}^{61} \pi(61^2 - y^2)\,dy$

$= \ldots$

equation of circle

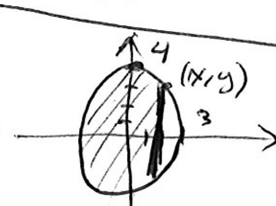
$x^2 + y^2 = 61^2$

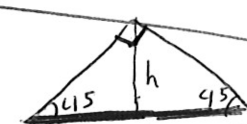
$x = \sqrt{61^2 - y^2}$

Volume= ______

10. (2 pts) UNCC1242/EssentialCalculus-Stewart-Sec7.2.33.pg

Book Problem 33





The base of a certain solid is an elliptical region with boundary curve $16x^2 + 9y^2 = 144$. Cross-sections perpendicular to the x-axis are isosceles right triangles with hypotenuse in the base.

$b = 2y = 2\sqrt{\frac{144-16x^2}{9}}$

$h = \frac{b}{2} = \sqrt{\frac{144-16x^2}{9}}$

Use the formula $V = \int_a^b A(x)\,dx$ to find the volume of the solid.

$V = \int_{-3}^{3} A(x)\,dx$ with $A(x) = \frac{1}{2}bh = \frac{1}{2} \cdot 2\sqrt{\ } \cdot \sqrt{\ }$

$A(x) = \frac{144-16x^2}{9}$

The lower limit of integration is $a =$ -3

The upper limit of integration is $b =$ 3

The base of the triangular cross-section is the following function of x: $2\sqrt{\frac{144-16x^2}{9}}$

The height of the triangular cross-section is the following function of x: $\sqrt{\frac{144-16x^2}{9}}$

The area of the triangular cross-section is $A(x)=$ $\frac{144-16x^2}{9}$

$V = \int_{-3}^{3} \ldots$

Thus the volume of the solid is $V =$ ______

11. (2 pts) UNCC1242/EssentialCalculus-Stewart-Sec7.2.35.pg

Book Problem 35 $x^2 = \frac{y}{36}$

The base of a certain solid is the area bounded above by the graph of $y = 25$ and below by the graph of $y = 36x^2$. Cross-sections perpendicular to the y-axis are squares.

Use the formula $V = \int_a^b A(y)\,dy$ to find the volume of the solid.

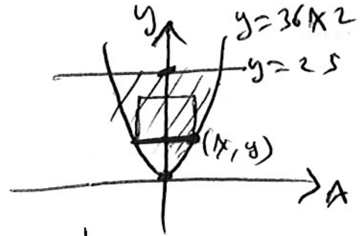
The lower limit of integration is $a =$ 0

The upper limit of integration is $b =$ 25

The side s of the square cross-section is the following function of y: $2\sqrt{\frac{y}{36}} = \frac{2\sqrt{y}}{6} = \sqrt{y}/3$

The area of the square cross-section is $A(y)=$ $y/9$

Thus the volume of the solid is $V =$ ...



$A(y) = s^2 = (2x)^2$

$= 4x^2 = 4\left(\frac{y}{36}\right)$

$A(y) = \frac{y}{9}$

$V = \int_a^b A(y)dy$

$V = \int_0^{25} \left(\frac{y}{9}\right) dy$

$V = \frac{y^2}{18}\Big|_0^{25} = \frac{25^2}{18} = \ldots$

$s = 2x = 2 \cdot \sqrt{\frac{y}{36}}$

12. (2 pts) UNCC1242/EssentialCalculus-Stewart-Sec7.2.37.pg

Book Problem 37

The base of a certain solid is the triangle with vertices (0,0), (4,0), and (0,3). Cross-sections perpendicular to the y-axis are isosceles triangles with height equal to the base.

Use the formula $V = \int_a^b A(y)\,dy$ to find the volume of the solid.

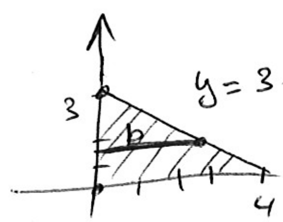
The lower limit of integration is $a =$ 0

The upper limit of integration is $b =$ 3

The base of the triangular cross-section is the following function of y: $\frac{4}{3}(3-y)$

The area of the triangular cross-section is $A(y)=$ $\frac{8}{9}(3-y)^2$

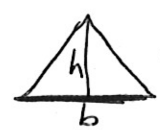
Thus the volume of the solid is $V =$ ...



$y = 3 - \frac{3}{4}x$ (equation of the line)

$b = x = \frac{4}{3}(3-y)$

solve for x



with $h=b$

$A(y) = \frac{1}{2}bh = \frac{1}{2}b^2$

$A(y) = \frac{1}{2}\left[\frac{4}{3}(3-y)\right]^2$

$= \frac{1}{2} \cdot \frac{16}{9}(3-y)^2$

$= \frac{8}{9}(3-y)^2$

$V = \int A(y)\,dy = \int_0^3 \frac{8}{9}(3-y)^2 dy$

$= \frac{8}{9}\int_0^3 (9 - 6y + y^2)dy$

$= \frac{8}{9}\left(9y - 3y^2 + \frac{y^3}{3}\right)\Big|_0^3$

$= \ldots$