Wafaa Shaban **Fall2010-Math1242Common 005**

WeBWorK assignment number Sec7.3 is due : 10/21/2010 at 11:30pm EDT.

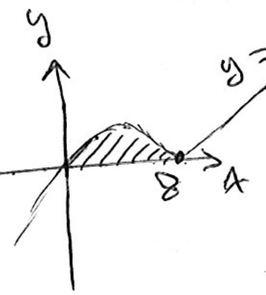
An alternative method to Disks and Washers is the Method of Cylindrical Shells. Some problems that are difficult for Disks are easy for Shells. In some cases, when both methods are easy, having two ways to solve a problem offers a way to check your results, for example on an exam.

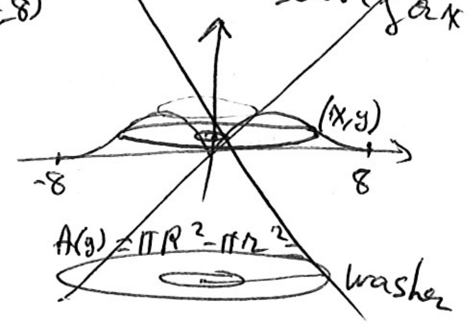
The Method of Cylindrical Shells:

The volume of a solid, obtained by rotating about the y-axis the region under the curve $y = f(x)$ from a to b, is $V = \int_a^b 2\pi x f(x)dx$.

Note that $2\pi x$ = the circumference of the cylindrical shell and $f(x)$ = the height of that shell.

Thus $V = \int_a^b$ (circumference)(height) dx.





1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.1.pg

Book Problem 1

Let S be the solid obtained by rotating the region bounded by the given curves about the y-axis: $y = x(x-8)^2$, $y = 0$. Note that it is akward to use the method of slicing to find the volume V of S. Sketch a typical approximating shell.

x-axis

$$V = \int_0^8 A(x)\,dx$$

The circumference of this shell = $2\pi x$

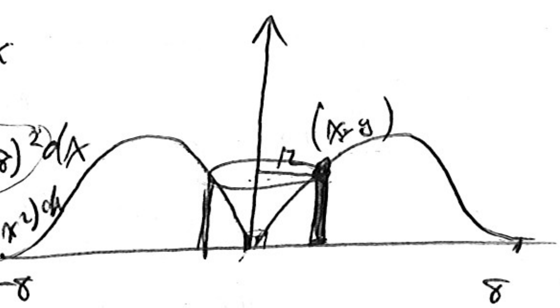
The height of this shell = $x(x-8)^2$

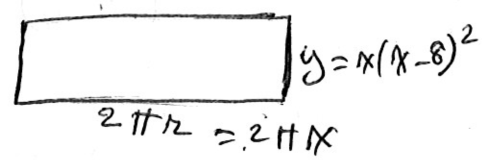
Using the method of cylindrical shells, V = $\int_a^b$ ___ dx, where $a = 0$ and $b = 8$.

$$V = \int_0^8 2\pi x^2(x-8)^2\,dx$$

$$= 2\pi\int_0^8 (x^4 - 16x^3 + 64x^2)\,dx$$

Therefore the volume V = $2\pi\left(\frac{8^5}{5} - 16\frac{8^4}{4} + 64\frac{8^3}{3}\right)$





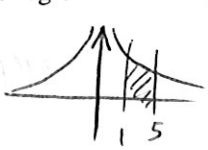
$$A(x) = 2\pi x \cdot x(x-8)^2$$

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.3.pg

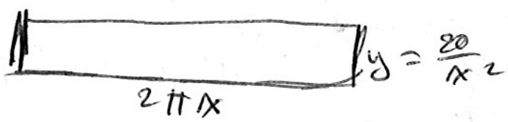
Book Problem 3

Use the method of cylindrical shells to find the volume V of the solid obtained by rotating the region bounded by the given curves about the y-axis:

$y = 20/x^2$, $y = 0$, $x = 1$, $x = 5$;



The circumference of a typical shell = $2\pi x$ and the height of this shell = $\frac{20}{x^2}$

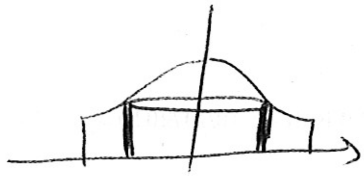


Therefore the volume V of the solid = $\int_1^5 \frac{40\pi}{x}\,dx$ =

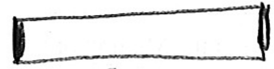
$$A(x) = 2\pi x \cdot \frac{20}{x^2} = \frac{40\pi}{x}$$

$$V = \int_1^5 \frac{40\pi}{x} = 40\pi \ln|x|\Big|_1^5 = \boxed{40\pi \ln 5} = 202.24793 2$$

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.5.pg



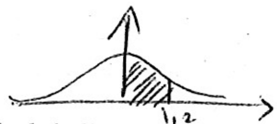
$y = 2e^{-x^2}$



$2\pi x$

$A(x) = 2\pi x \, 2e^{-x^2} = 4\pi x e^{-x^2}$

Book Problem 5



Use the method of cylindrical shells to find the volume V of the solid obtained by rotating the region bounded by the given curves about the y-axis:

$y = 2e^{-x^2}, y = 0, x = 0, x = 1.2;$

$V = \int_0^{1.2} 4\pi x e^{-x^2}$ Let $u = -x^2 \Rightarrow du = -2x\,dx$

$= \int 4\pi x e^{u} \frac{du}{-2x} = -2\pi \int e^u du = -2\pi e^{-x^2} \Big/_0^{1.2}$

$= \boxed{-2\pi e^{-(1.2)^2} + 2\pi e^{0}}$

The circumference of a typical shell = $2\pi x$
The height of this shell = $2e^{-x^2}$

Thus the volume V = $\int_0^{1.2} 4\pi x e^{-x^2} \, dx = 4.7945$

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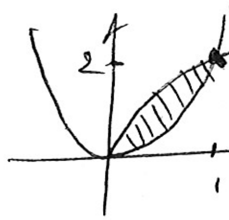
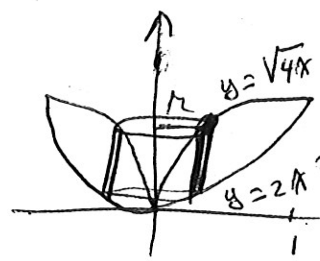
Book Problem 8



$2x^2 = \sqrt{4x}$
$4x^4 = 4x$
$x^3 = 1$

Let V be the volume of the solid S obtained by rotating about the y-axis the region bounded by $y = \sqrt{4x}$ and $y = 2x^2$. Find V both by slicing and by cylindical shells:



(A) The method of cylindrical shells :

The circumference of a typical shell = $2\pi x$ and the height of this shell = $\sqrt{4x} - 2x^2$

The volume V = $\int_a^b 2\pi x(\sqrt{4x} - 2x^2) \, dx$, where $a = 0$ and $b = 1$.

Therefore V = $4\pi\left(\frac{2}{5} - \frac{1}{4}\right) = 1.8849556$

$A(x) = 2\pi x(\sqrt{4x} - 2x$

$V = \int_0^1 A(x)dx = 2\pi \int_0^1 (2x\sqrt{x} - 2$

$= 4\pi \int_0^1 (x^{3/2} - x^3) dx = 4\pi\left(\frac{2}{5}x^{5/2} - \right.$

$= 4\pi\left(\frac{2}{5} - \frac{1}{4}\right) = \ldots$

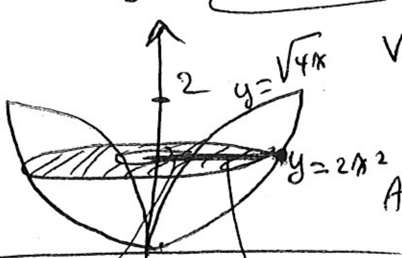
(B) The method of slicing from Sec(7.2):

The volume V = $\int_a^b \pi\left(\frac{y}{2} - y^4/16\right) dy$, where $a = 0$ and $b = 2$.

Thus the volume V = ______.

$y = \sqrt{4x} \Rightarrow 4x = y^2 \Rightarrow \boxed{x = y^2/4}$

$y = 2x^2$
$x^2 = \frac{y}{2}$
$\boxed{x = \sqrt{\frac{y}{2}}}$



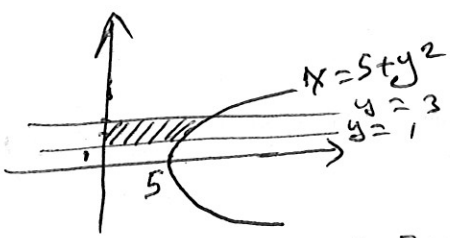
$r = \frac{y^2}{4}$ $R = \sqrt{\frac{y}{2}}$

$V = \int_0^2 A(y)dy =$

$A(y) = \pi R^2 - \pi r^2$

$= \pi\left(\sqrt{\frac{y}{2}}\right)^2 - \pi\left(\frac{y^2}{4}\right)^2 = \pi\left(\frac{y}{2} - \frac{y^4}{16}\right) = A(y)$

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.9.pg



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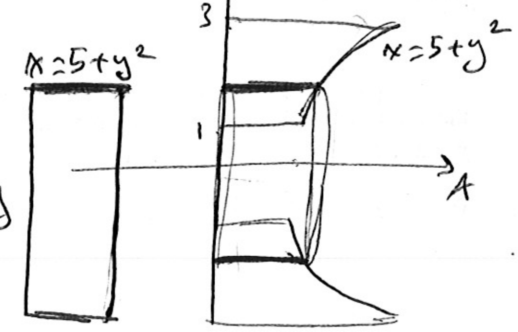
Book Problem 9

$y = \pm\sqrt{x-5}$

Use the method of cylindrical shells to find the volume V of the solid obtained by rotating the region bounded by the given curves about the x-axis:

$x = 5+y^2\,, x = 0\,, y = 1\,, y = 3\,;$



$\boxed{A(y) = 2\pi y(5+y^2)}$ $2\pi y$

$V = \int_1^3 A(y)\,dy$

$= 2\pi \int_1^3 (5y + y^3)\,dy = 2\pi\left(\frac{5y^2}{2} + \frac{y^4}{4}\right)\Big|_1^3$

$= 2\pi\left(\left(\frac{45}{2} + \frac{81}{4}\right) - \left(\frac{5}{2} + \frac{1}{4}\right)\right) = 2\pi\left(\cancel{22.5} + \frac{45}{\ldots} - \cancel{2.5}\,\frac{5}{3}\right)$

$= 2\pi\left(40 \ldots \frac{5}{3}\right) = 80\pi$

The circumference of a typical shell in terms of y = $2\pi y$
The height of this shell in terms of y = $\cancel{5+y^2}$

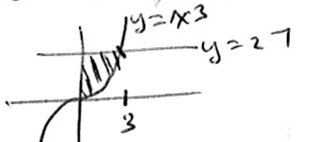
Thus the volume V = $\int_1^3 2\pi y(5+y^2)\,dy$ = 251.32741

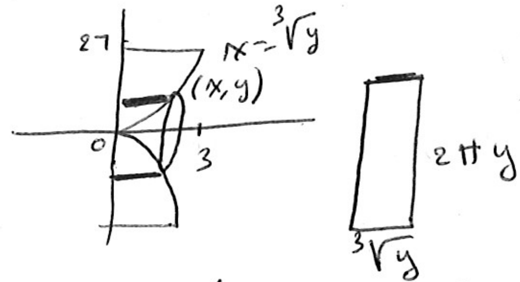
6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.11.pg

Book Problem 11

(A) Use the method of cylindrical shells to find the volume V of the solid S obtained by rotating the region bounded by the given curves about the x-axis:

$y = x^3\,, x = 0\,, y = 27\,;$





The circumference of a typical shell in terms of y = $2\pi y$
The height of this shell in terms of y = $\sqrt[3]{y}$

The volume V = $\int_a^b 2\pi y^{4/3}\,dy$, where $a =$ 0 and $b =$ 27

$A(y) = 2\pi y \cdot \sqrt[3]{y} = 2\pi y^{4/3}$

$V = \int_0^{27} 2\pi y^{4/3}\,dy = 2\pi \frac{3}{7} y^{7/3}\Big|_0^{27}$

$= \frac{6\pi}{7}(27)^{7/3} = \frac{6\pi}{7}(3)^7$

Therefore V = $\frac{6\pi}{7}(3)^7 = 5889.139829$

(B) Use the method of slicing to find the volume V of the solid S:

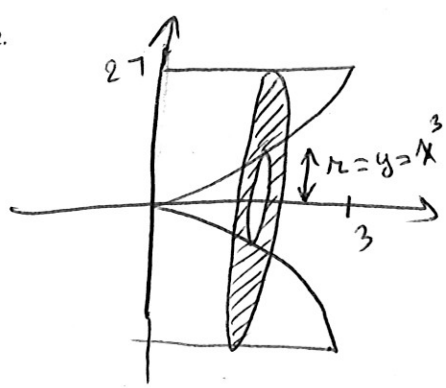
The volume V = $\int_a^b \pi(27^2 - x^6)\,dx$, where $a =$ 0 and $b =$ 3.

Thus the volume V = $\pi\left(27^2x - \frac{x^7}{7}\right)\Big|_0^3$

$= \pi\left(27^2 \cdot 3 - \frac{3^7}{7}\right)$

$= \ldots\ldots$

same as above

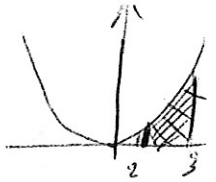


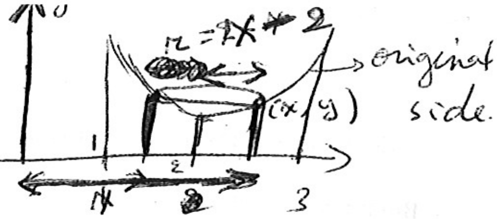
$A(x) = \pi R^2 - \pi r^2$

$= \pi(27)^2 - \pi(x^3)^2$

$= \pi[27^2 - x^6]$

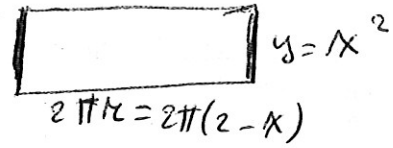
7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.15.pg





Book Problem 15

Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2, x = 2, x = 3$; about the line $x = 2$.



V = 22.514747

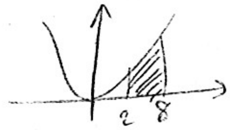
$$A(x) = 2\pi(x-2)x^2$$

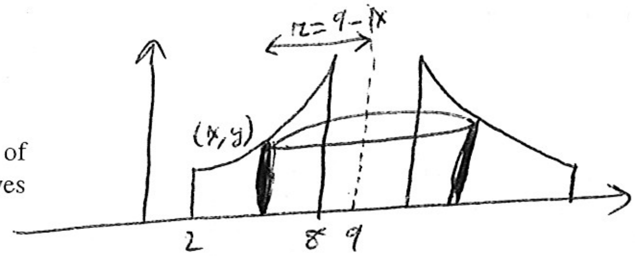
$$V = \int_1^3 2\pi x^2(x-2)\,dx = 2\pi\int_2^3 (-2x^2 + x^3)\,dx = 2\pi\left(\frac{-2x^3}{3} + \frac{x^4}{4}\right)\Big|_2^3$$

$$= 2\pi\left(-18 + \frac{81}{4}\right) - 2\pi\left(\frac{-16}{3} + \frac{16}{4}\right) = \boxed{\ldots\ldots\ldots}$$

8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.17.pg

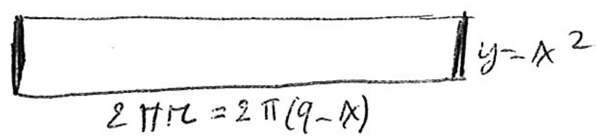
Book Problem 17





Use the method of cylindrical shells to find the volume of the solid obtained by rotating the region bounded by the curves $y = x^2, x = 2, x = 8$; about the line $x = 9$.

V = $\int_2^8$ $2\pi(9-x)x^2$ dx = $2\pi(492)$ = 3091.327



$$= 2\pi\int_2^8 (9x^2 - x^3)\,dx$$

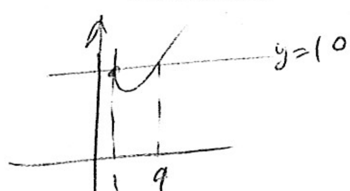
$$= 2\pi\left(3x^3 - \frac{x^4}{4}\right)\Big|_2^8 = 2\pi\left(3(8^3) - \frac{8^4}{4}\right) - 2\pi(24 - 4)$$

$$= \ldots$$

$$A(x) = 2\pi(9-x)x^2$$

9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.3.35.pg

Book Problem 35



intersection : $x + \frac{9}{x} = 10$

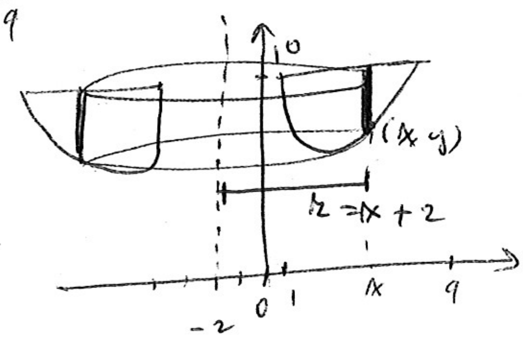
$x^2 + 9 = 10x$

$x = 1$ or 9

Let S be the solid obtained by rotating the region bounded by the curves $y = x + (9/x)$, $y = 10$, about $x = -2$.

The volume V = $\int_a^b$ $A(x)$ dx, where $a =$ 1 and $b =$ 9.

Therefore the volume V = $251.5666\pi = \boxed{790.31973}$



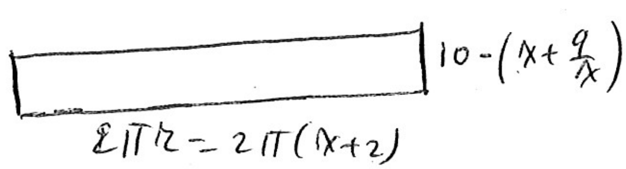
$$V = \int_1^9 2\pi(x+2)\left(10 - x - \frac{9}{x}\right)dx$$

$$V = 2\pi\int\left(10x - x^2 - 9 + 20 - 2x - \frac{18}{x}\right)dx$$

$$= 2\pi\int\left(8x - x^2 + 11 - \frac{18}{x}\right)dx$$

$$= 2\pi\left(4x^2 - \frac{x^3}{3} + 11x - 18\ln|x|\right)\Big|_1^9$$

$$= \ldots\ldots$$



$$A(x) = 2\pi(x+2)\left(10 - x - \frac{9}{x}\right)$$