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Fall2010-Math1242Common 005

WeBWorK assignment number Sec7.4 is due : 10/22/2010 at 11:30pm EDT.

The Arc Length Formula

The length of a curve with equation $y = f(x), a \le x \le b$ is $L = \int_a^b \sqrt{1+(y')^2}dx$

The length of a curve with equation $x = f(y), a \le y \le b$ is $L = \int_a^b \sqrt{1+(x')^2}dy$

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1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.3.pg

1 **Book Problem 3**

Set up an integral to find the length of the curve defined by $y = 5x^{3/2} + 3$ from $x = 1$ to $x = 10$, then evaluate it.

$L = \int_1^{10}$ ________ $dx =$ ____________

$y = 5x^{3/2} + 3$

$y' = 5 \cdot \frac{3}{2} x^{1/2} = \frac{15}{2}\sqrt{x}$

$L = \int_1^{10} \sqrt{1+\left(\frac{15}{2}\sqrt{x}\right)^2}\,dx$

$= \int_1^{10} \sqrt{1+\frac{225}{4}x}\,dx$

$= \left(\frac{4}{225}\right)\frac{2}{3}\left(1+\frac{225}{4}x\right)^{3/2}\Big/_1^{10}$

$= \dots$

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.5.pg

2 **Book Problem 5**

Consider the curve defined by $y = \frac{x^6}{8} + \frac{1}{12x^4}$ from $x = 1$ to $x = 3$.

The length of this curve is $L = \int_1^3 \sqrt{1+(f'(x))^2}\,dx$ where $f'(x) =$ $\frac{3}{4}x^5 - \frac{1}{3x^5}$

Simplify and factor to get $L = \int_1^3 \sqrt{(g(x))^2}\,dx$ where $g(x) =$ $\frac{3}{4}x^5 + \frac{1}{3x^5}$

Simplify and integrate to find L = ____________ .

$y' = \frac{6x^5}{8} + \frac{-4x^{-5}}{12} = \frac{3}{4}x^5 - \frac{1}{3x^5} = y'$

$L = \int_1^3 \sqrt{1+\left(\frac{3}{4}x^5 - \frac{1}{3x^5}\right)^2}\,dx$

$= \int_1^3 \sqrt{1+\frac{9}{16}x^{10} - 2 \cdot \frac{3}{4}x^5 \cdot \frac{1}{3x^5} + \frac{1}{9x^{10}}}\,dx$

$= \int_1^3 \sqrt{1+\frac{9}{16}x^{10} - \frac{1}{2} + \frac{1}{9x^{10}}}\,dx$

$= \int_1^3 \sqrt{\frac{9}{16}x^{10} + \frac{1}{2} + \frac{1}{9x^{10}}}\,dx$

$= \int_1^3 \sqrt{\left(\frac{3}{4}x^5 + \frac{1}{3x^5}\right)^2}\,dx$

$= \int_1^3 \left(\frac{3}{4}x^5 + \frac{1}{3x^5}\right)dx =$

$= \frac{3}{4}\frac{x^6}{6} + \frac{-1}{12x^4}\Big/_1^3 = \dots$

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.6.pg

Book Problem 6

$y' = \frac{1}{8}\left(-8x + \frac{2}{x}\right) = -x + \frac{1}{4x}$

Find the length L of the arc formed by $y = \frac{1}{8}\left(-4x^2 + 2\ln(x)\right)$ from $x = 3$ to $x = 5$

Set up: $L = \int_3^5 \sqrt{1+(f'(x))^2}\,dx$ where $f'(x) =$ $-x + \frac{1}{4x}$

Simplify: $L = \int_3^5 \sqrt{(g(x))^2}\,dx$ where $g(x) =$ $x + \frac{1}{4x}$

Integrate: L = ____________ .

$L = \int \sqrt{1+\left(-x+\frac{1}{4x}\right)^2}\,dx$

$= \int \sqrt{1+x^2 - 2x\frac{1}{4x} + \frac{1}{16x^2}}\,dx$

$= \int \sqrt{1+x^2 - \frac{1}{2} + \frac{1}{16x^2}}\,dx$

$= \int \sqrt{x^2 + \frac{1}{2} + \frac{1}{16x^2}}\,dx = \int \sqrt{\left(x + \frac{1}{4x}\right)^2}\,dx = \int_3^5 \left(x + \frac{1}{4x}\right)dx = \frac{x^2}{2} + \frac{\ln|x|}{4}\Big/_3^5 = \frac{25}{2} + \frac{\ln 5}{4} - \left(\frac{9}{2} + \frac{\ln 3}{4}\right) = \dots$

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.7.pg

Book Problem 7

$$L = \int_0^{243} \sqrt{1+(x')^2}\,dy$$

Find the length L of the curve $x = \sqrt[5]{y}\left(y - \frac{25}{96}\sqrt[5]{y^3}\right)$, $0 \le y \le 243$.

Set up: $L = \int_0^{243} \sqrt{1+(f'(y))^2}\,dy$ where $f'(y) = \frac{6}{5}y^{1/5} - \frac{5}{24}y^{-1/5}$

Simplify: $L = \int_0^{243} \sqrt{(g(y))^2}\,dy$ where $g(y) = \frac{6}{5}y^{1/5} + \frac{5}{24}y^{-1/5}$

Integrate: $L = (243)^{6/5} + \frac{25}{96}(243)^{4/5} = 111$

$x = y^{6/5} - \frac{25}{96}y^{4/5} \Rightarrow x' = \frac{6}{5}y^{1/5} - \frac{25}{96}\cdot\frac{4}{5}y^{-1/5}$

$x' = \frac{6}{5}y^{1/5} - \frac{5}{24}y^{-1/5}$

$$L = \int_0^{243}\sqrt{1+\left(\frac{6}{5}y^{1/5} - \frac{5}{24}y^{-1/5}\right)^2}\,dy$$

$$= \int_0^{243}\sqrt{1+\frac{36}{25}y^{2/5} - 2\cdot\frac{6}{5}y^{1/5}\cdot\frac{5}{24}y^{-1/5} + \frac{25}{24^2}y^{-2/5}}$$

$$= \int_0^{243}\sqrt{1+\frac{36}{25}y^{2/5} - \frac{1}{2} + \frac{25}{24^2}y^{-2/5}}\,dy$$

$$= \int_0^{243}\sqrt{\frac{36}{25}y^{2/5} + \frac{1}{2} + \frac{25}{24^2}y^{-2/5}}\,dy$$

$$= \int_0^{243}\sqrt{\left(\frac{6}{5}y^{1/5} + \frac{5}{24}y^{-1/5}\right)^2}\,dy = \int_0^{243}\left(\frac{6}{5}y^{1/5} + \frac{5}{24}y^{-1/5}\right)dy$$

$$= \frac{6}{5}\cdot\frac{5}{6}y^{6/5} + \frac{5}{24}\cdot\frac{5}{4}y^{4/5}\Big|_0^{243} = y^{6/5} + \frac{25}{96}y^{4/5}\Big|_0^{243}$$

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5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.9.pg

Book Problem 9

$y = \ln(\sec x) = \ln\left(\frac{1}{\cos x}\right) = \underbrace{\ln 1}_{0} - \ln(\cos x)$

Find the length L of the arc formed by $y = \ln(\sec x)$, $0 \le x \le \pi/3$.

$y = -\ln(\cos x)$

$L = \int_0^{\pi/3} \sqrt{1+(f'(x))^2}\,dx$ where $f'(x) = \tan x$

$L = \int_0^{\pi/3} \sqrt{(g(x))^2}\,dx$ where $g(x) = \sec x$

Now use the Table of Integrals at the end of your book to evaluate L:

Formula number 14 and the length L of the curve = 1.3169579.

$y' = -\frac{(\cos x)'}{\cos x} = -\frac{-\sin x}{\cos x} = \tan x$

$L = \int \sqrt{1+\tan^2 x}\,dx = \int\sqrt{\sec^2 x}\,dx$

trig.

$L = \int_0^{\pi/3} \sec x\,dx = \ln|\sec x + \tan x|\Big|_0^{\pi/3}$

$\boxed{14}$ table

$= \ln\left|\sec\frac{\pi}{3} + \tan\frac{\pi}{3}\right| - \ln|\sec 0 + \tan 0|$

$= \boxed{1.3169579}$

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6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.13.pg

Book Problem 13

$$L = \int_0^3 \sqrt{1+y'^2}\,dx$$

Find the length L of the arc formed by $y = e^{2x}/2$, $0 \le x \le 3$.

$y' = \frac{2e^{2x}}{2} = e^{2x}$

$L = \int_0^3 \sqrt{1+e^{4x}}\,dx.$

$$L = \int_0^3 \sqrt{1+(e^{2x})^2}\,dx$$

Evaluate L using the Table of Integrals at the end of your book:

First, perform the substitution $u = e^{2x}$ $\Rightarrow du = 2e^{2x}dx \Rightarrow dx = \frac{du}{2e^{2x}}$, (Hint: $u^2 = e^{4x}$)

$x = 0 \Rightarrow u = e^0 = 1$

$x = 3 \Rightarrow u = e^6$

to get $L = \int_1^a \frac{\sqrt{1+u^2}}{2u}\,du$.

Then use formula number 23 to evaluate L = ________.

$$L = \int_0^3 \sqrt{1+e^{4x}}\,dx = \int\sqrt{1+u^2}\,\frac{du}{2e^{2x}} = \frac{1}{2}\int\frac{\sqrt{1+u^2}}{u}\,du$$

$$= \int_1^{e^6}\frac{\sqrt{1+u^2}}{2u}\,du = \frac{1}{2}\left[\sqrt{1+u^2} - 1\ln\left|\frac{1+\sqrt{1+u^2}}{u}\right|\right]_1^{e^6}$$

$$= \frac{1}{2}\left[\sqrt{1+e^{12}} - \ln\left|\frac{1+\sqrt{1+e^{12}}}{e^6}\right|\right] - \frac{1}{2}\left[\sqrt{2} - \ln|1+\sqrt{2}|\right]$$

7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.19.pg

$y' = 2\cdot(-2)e^{-2x} = -4e^{-2x}$

Book Problem 19

Use Simpson's Rule with $n = 4$ to estimate the arc length of the curve $y = 2e^{-2x}$, $\quad 0 \le x \le 2$.

L = $\int_0^2 f(x)dx$ where $f(x) =$ ~~$\sqrt{1+e^{-4x}}$~~ $\sqrt{1+16e^{-4x}}$

The estimation $S_4 =$ 3.099195

$$L = \int_0^2 \sqrt{1+y'^2}\,dx = \int_0^2 \sqrt{1+(4e^{-2x})^2}\,dx$$

$$L = \int_0^2 \sqrt{1+16e^{-4x}}\,dx$$

0 .5 1 1.5 2

$\Delta x = \frac{2-0}{4} = \frac{1}{2}$

$$\approx S_4 = \frac{\Delta x}{3}\left[f(0)+4f(.5)+2f(1)+4f(1.5)+f(2)\right]$$

$$= \frac{.5}{3}\left[\sqrt{17}+4\sqrt{1+16e^{-2}}+2\sqrt{1+16e^{-4}}+4\sqrt{1+16e^{-6}}+\sqrt{1+16e^{-8}}\right.$$

$= \ldots\ldots$

Or use program $Y_1 = \sqrt{1+16e^{-4x}}$

$A=0,\ B=2,\ N=2$

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8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.4.29.pg

Book Problem 29

$y' = -\frac{2x}{42} = \frac{-x}{21}$

A hawk flying at 14 m/s at an altitude of 180 m accidentally drops its prey. The parabolic trajectory of the falling prey is described by the equation $y = 180 - x^2/42$ until it hits the ground, where y is the height above the ground and x is the horizontal distance traveled in meters.

hits the ground when $y = 0 = 180 - \frac{x^2}{42}$

$\Rightarrow \frac{x^2}{42} = 180 \Rightarrow x = \sqrt{180 * 42} = 86.94826$

Let D be the distance traveled by the prey from the time it is dropped until the time it hits the ground.

D = $\int_a^b \sqrt{1+x^2/441}\,dx$, where $a =$ 0 and $b =$ 86.94826

Therefore the distance traveled by the prey is equal to ____.

$$D = L = \int \sqrt{1+y'^2}\,dx$$

$$= \int_0^{86.94826} \sqrt{1+\left(\frac{-x}{21}\right)^2}\,dx$$

$$= \int_0^{86.9\ldots} \sqrt{1+\frac{x^2}{441}}\,dx$$

Math 9 $= 207.521821$