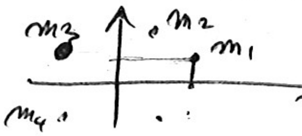
Consider a system of n particles with masses $m_1, m_2, \ldots, m_n$ at the points $(x_1,y_1),(x_2,y_2),\ldots,(x_n,y_n)$. Its **center of mass** is located at Total mass $m = m_1 + m_2 + \cdots + m_n$

$$\bar{x} = \frac{m_1x_1 + m_2x_2 + \cdots + m_nx_n}{m}$$

$$\bar{x} = \frac{\sum_{i=1}^n m_i x_i}{\sum_{i=1}^n m_i}, \ \bar{y} = \frac{\sum_{i=1}^n m_i y_i}{\sum_{i=1}^n m_i}.$$

$= m$



$M_y = \sum m_i x_i$

$M_x = \sum m_i y_i$

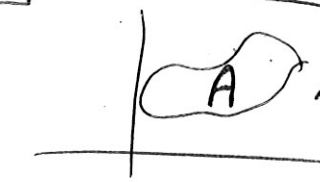
Let R be the flat region with density ρ under the curve $y = f(x)$ for $a \le x \le b$, then the **Moments** of R about the y-axis and x-axis are:

$$\boxed{M_y = \rho \int_a^b x f(x)dx, \ \bar{M}_x = \rho \int_a^b \frac{1}{2}[f(x)]^2 dx.}$$

$$\boxed{\bar{x} = \frac{M_y}{m}, \ \bar{y} = \frac{M_x}{m}}$$

The **Center of Mass** or the **Centroid** of the region is located at the point $(\bar{x},\bar{y})$, where

$$\bar{x} = \frac{1}{A}\int_a^b x f(x)dx, \ \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}[f(x)]^2 dx,$$



$m = \rho A$

where A is the area of the region , $A = \int_a^b f(x)dx$. $= \int_a^b (y_{top} - y_{bottom})dx$

If the region R lies between two curves $y = f(x)$ and $y = g(x)$, where $f(x) \ge g(x)$ for $a \le x \le b$, then the **Center of Mass** of the region is located at the point $(\bar{x},\bar{y})$; where

$$\bar{x} = \frac{1}{A}\int_a^b x(f(x) - g(x))dx, \ \bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}([f(x)]^2 - [g(x)]^2)dx.$$

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1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.33.pg

Book Problem 33

The masses m_i are located at the points P_i. Find the moments M_x and M_y and center of mass of the system.

$m_1 = 4, \ m_2 = 2, \ m_3 = 3.$

$P_1 = (2,7), \ P_2 = (3,4), \ P_3 = (-9,-1).$

$M_x =$ 33

$M_y =$ -13

$\bar{x} =$ -13/9

$\bar{y} =$ 11/3

total mass $m = m_1 + m_2 + m_3 = 4 + 2 + 3 = 9$

$M_x = \sum m_i y_i = m_1 y_1 + m_2 y_2 + m_3 y_3$
$= 4(7) + 2(4) + 3(-1) = 33$

$M_y = \sum m_i x_i = m_1 x_1 + m_2 x_2 + m_3 x_3$
$= 4(2) + 2(3) + 3(-9) = -13$

$\bar{x} = \frac{M_y}{m} = \frac{-13}{9}$, $\bar{y} = \frac{M_x}{m} = \frac{33}{9} = \frac{11}{3}$

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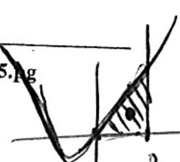
2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.35.pg



Book Problem 35

Find the centroid $(\bar{x},\bar{y})$ of the region bounded by:

$8x^2 + 4x$, $y = 0$ (x-axis), $x = 0$ (y-axis), and $x = 2$.

$\bar{x} = \frac{1}{A}\int_0^2 x f(x)dx = \frac{1}{\frac{88}{3}}\int_0^2 x(8x^2 + 4x)dx$

$A = \int_0^2 (8x^2 + 4x)dx$
$= \frac{8x^3}{3} + 2x^2 \Big|_0^2$
$= \frac{64}{3} + 8 = \frac{88}{3}$

$\bar{y} = \frac{1}{A}\int_0^2 \frac{1}{2}[f(x)]^2 dx$
$= \frac{1}{\frac{88}{3}}\int_0^2 \frac{1}{2}(8x^2 + 4x)^2 dx$
$= \frac{3}{88}\left(\frac{1}{2}\right)\int_0^2 (64x^4 + 64x^3 + 16x^2)dx$

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.37.pg

Book Problem 37

Find the centroid $(\bar{x}, \bar{y})$ of the region bounded by:

$y = e^{2x}$, $y = 0$, $x = 0$, and $x = 4$.

$\bar{x} =$ ______________

$\bar{y} =$ ______________

---

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.39.pg

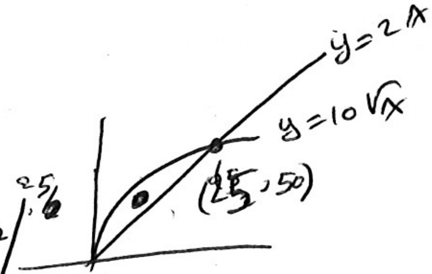
Book Problem 39

$$A = \int_0^{25} (10\sqrt{x} - 2x)\, dx = \frac{20}{3} x^{3/2} - x^2 \Big|_0^{25} = 208.333\ldots$$

Find the centroid $(\bar{x}, \bar{y})$ of the region bounded by the two curves $y = 10\sqrt{x}$ and $y = 2x$.

$\bar{x} =$ ______________

$\bar{y} =$ ______________



intersection

$2x = 10\sqrt{x}$

$4x^2 = 100x$

$x = 25$

$$\bar{x} = \frac{1}{A}\int x\left[f(x) - g(x)\right]dx = \frac{1}{208.333}\int_0^{25} x\left(10\sqrt{x} - 2x\right)dx$$

$$= \frac{1}{208.33333}\int_0^{25}\left(10x^{3/2} - 2x^2\right)dx = \ldots$$

$$\bar{y} = \frac{1}{A}\int \frac{1}{2}\left[(f(x))^2 - (g(x))^2\right]dx$$

$$= \frac{1}{208.333}\left(\frac{1}{2}\right)\int_0^{25}\left(100x - 4x^2\right)dx = \ldots$$

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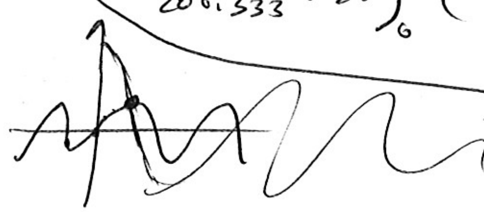
5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.41.pg

Book Problem 41

Find the centroid $(\bar{x}, \bar{y})$ of the region bounded by the two curves $y = \sin x$, $y = 4\cos x$, $x = 0$, and $x = \pi/3$.

$\bar{x} =$ ______________

$\bar{y} =$ ______________

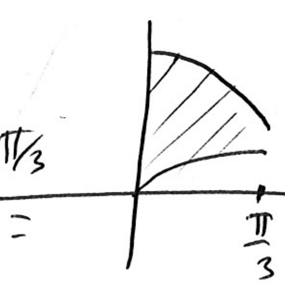


intersection $x = 1.3258$.

$\frac{\pi}{3} = 1.04$

$$\bar{x} = \frac{1}{A}\int_0^{\pi/3} x\left(f(x) - g(x)\right)dx$$

$$= \frac{1}{2.964}\int_0^{\pi/3} x\left(4\cos x - \sin x\right)dx$$



$$A = \int_0^{\pi/3}\left(4\cos x - \sin x\right)dx = 2.964$$

$$\int_0^{\pi/3} x\cos x\, dx = x\sin x - \int \sin x\, dx = x\sin x + \cos x \Big|_0^{\pi/3} =$$

integration by parts: let $u = x \rightarrow du = dx$

$dv = \cos x\, dx \rightarrow v = \sin x$

$$\bar{y} = \frac{1}{A}\int_0^{\pi/3} \frac{1}{2}\left[16\cos^2 x - \sin^2 x\right]dx = \text{Math 9} = \text{table} = \ldots$$

6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.43.pg

Book Problem 43

A lamina has the shape of a triangle with vertices at $(-9,0)$, $(9,0)$, and $(0,7)$. Its density is $\rho = 9$.

A. What is the total mass? ________

B. What is the moment about the x-axis? ________

C. What is the moment about the y-axis? ________

D. Where is the center of mass? (________, ________)

7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.44.pg

Book Problem 44

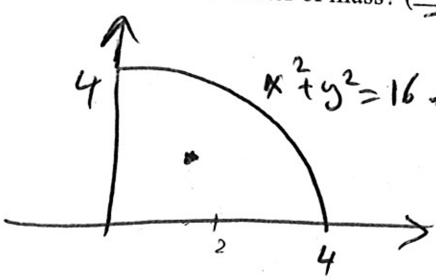
A lamina occupies the part of the disk $x^2 + y^2 \leq 16$ in the first quadrant. Its density is $\rho = 5$.

A. What is the total mass? 20π

B. What is the moment about the x-axis? $320/3$

C. What is the moment about the y-axis? $320/3$

D. Where is the center of mass? ($\frac{16}{3\pi}$, $\frac{16}{3\pi}$) $(1.6976, \ldots)$



$x^2+y^2=16 \Rightarrow y=\sqrt{16-x^2}$

A) total mass $m = \rho A = 5 \cdot \frac{1}{4}\pi r^2 = \frac{5}{4}\pi(4)^2 = \boxed{20\pi}$

B) $M_x = \rho\int_a^b \frac{1}{2}[f(x)]^2 dx = 5\left(\frac{1}{2}\right)\int_0^4 \left(\sqrt{16-x^2}\right)^2 dx = \frac{5}{2}\int_0^4 (16-x^2)dx = \frac{5}{2}\left(16x - \frac{x^3}{3}\right)\Big|_0^4$

$= \frac{5}{2}\left(64 - \frac{64}{3}\right) = \frac{5}{3}(64) = \frac{320}{3}$

C) $M_y = \rho\int_a^b x f(x) dx = 5\int_0^4 x\sqrt{16-x^2}\, dx$

Let $u = 16 - x^2$

$du = -2x\,dx \Rightarrow dx = \frac{du}{-2x}$

$= 5\int x\sqrt{u}\,\frac{du}{-2x} = -\frac{5}{2}\int u^{1/2} du$

$= -\frac{5}{2}\cdot\frac{2}{3}u^{3/2}\Big|_{\ldots}^{\ldots} = \frac{-5}{3}(16-x^2)^{3/2}\Big|_0^4 = \frac{-5}{3}(0) + \frac{5}{3}(16-0)^{3/2} = \frac{5}{3}(64)$

$= \frac{320}{3}$

d) $\bar{x} = \frac{M_y}{m} = \frac{320/3}{20\pi} = \frac{16}{3\pi}$

$\bar{y} = \frac{16}{3\pi}$