

$$W = \int f(x) dx$$

1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.1.pg

Book Problem 1

$$f(x) = 3x^3 - 3$$

A particle is moved along the x-axis by a force that measures $3x^3 - 3$ pounds at a point x feet from the origin. Find the work done in moving the particle from the origin to a distance of 2 feet.

Work = _____ ft-lb.

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.5.pg

Book Problem 5

$$f(x) = 3 \text{ lb}, x = \frac{6}{12} = 0.5 \text{ ft}$$

A force of 3 pounds is required to hold a spring stretched 6 in. beyond its natural length. How much work (in foot-pounds) is done in stretching the spring from its natural length to 12 in. beyond its natural length?

Work = $\int_a^b f(x) dx$, where $f(x) = \underline{\hspace{1cm}}$, $a = \underline{\hspace{1cm}}$, and $b = \underline{\hspace{1cm}}$.

Work = _____ ft-lb.

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Book Problem 6

cm \rightarrow m (divide by 100)

A spring has a natural length of 11 cm. If a 28-N force is required to keep it stretched to a length of 38 cm, how much work is required to stretch it from 11 cm to 26 cm?

Work = $\int_a^b f(x) dx$, where $f(x) = \underline{\hspace{1cm}}$, $a = \underline{\hspace{1cm}}$, and $b = \underline{\hspace{1cm}}$.

Work = _____ Joules.

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.7.pg

Book Problem 7

$$f(x) = Kx$$

Suppose that 3 Joules of work is needed to stretch a spring from its natural length of 14 cm to a length of 34 cm.

i) How much work is needed to stretch the spring from 20 cm to 30 cm? $\rightarrow x = 0.16$
 $W = \int_a^b f(x) dx$, where $f(x) = \underline{150x}$, $a = \underline{0.06}$, and $b = \underline{0.16}$
 $W = \underline{\hspace{2cm}} \text{ J.}$

ii) How far (in meters) beyond its natural length would a force of 35 N keep the spring stretched? $\underline{0.2333} \text{ m.}$

$$f(x) = 35 = 150x \rightarrow x = \frac{35}{150} = 0.2333$$

$$\text{work} = ft-lb$$

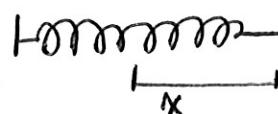
$$\text{work} = \text{Joules} = \text{Newton} \times \text{meter} = N \times m$$

$$W = \int_0^2 f(x) dx = \int_0^2 (3x^3 - 3) dx$$

$$= \frac{3x^4}{4} - 3x \Big|_0^2 = \dots \text{ft-lb}$$

Hook's law: $f(x) = Kx \rightarrow x$ is displacement beyond natural length

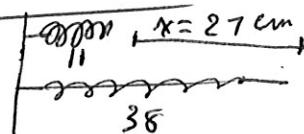
~~Force~~



$$3 = K(0.5) \Rightarrow K = 6$$

$$f(x) = 6x$$

$$W = \int_{x_{\text{beginning}}}^{x_{\text{end}}} f(x) dx = \int_0^1 6x dx = \dots$$



$$f(x) = Kx$$

$$28 = K \cdot (0.27)$$

$$K = \frac{28}{0.27} = 103.7037037$$

$$W = \int_0^{0.15} f(x) dx = \int_0^{0.15} 103.7037037x dx = \dots$$

$$W = 3 = \int_0^{0.20} Kx dx = \frac{Kx^2}{2} \Big|_0^{0.20} = \frac{K(0.20)^2}{2} = 0.02K$$

$$\Rightarrow K = \frac{3}{0.02} = 150$$

$x = 0.06$ because natural length = 14 cm

$$W = \int_{0.06}^{0.16} 150x dx = \frac{75x^2}{0.06} \Big|_{0.06}^{0.16} = \dots$$

Book Problem 9

A heavy rope, 60 ft long, weighs 0.9 lb/ft and hangs over the edge of a building 120 ft high.

- a) How much work is done in pulling the rope to the top of the building?

Work = 1620 ft-lb.

- a) How much work is done in pulling half the rope to the top of the building?

Work = _____ ft-lb.

Book Problem 11

A cable that weighs 3 lb/ft is used to lift 1200 lb of coal up a mine shaft 300 ft deep. Find the work done.

Work = _____ ft-lb.

Book Problem 15

An aquarium 13m long, 2m wide, and 7m deep is full of water. The density of water is 1000 kg/m^3 and the force of gravity is 9.8 m/s^2 .

- a) Find the work needed to pump all of the water out of the aquarium.

Work = $9800(13)(49)$ Joules. = 6242600

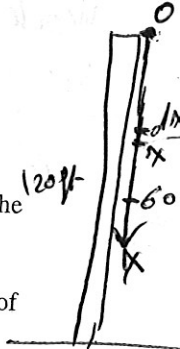
- b) Find the work needed to pump half of the water out of the aquarium.

Work = _____ Joules.

pump half: $W = \int_0^{3.5} 9800(26)y dy$

empty top half:

$= 9800(13)(3.5)^2$
 $= 1560650$



So to lift this slice

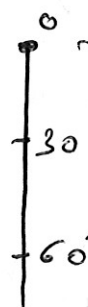
$\delta = 0.9 dx$

Work to lift this slice = $W = \text{force} \times \text{distance}$

$W = 0.9 dx \cdot x$

$W = 0.9x dx$

$W = \int_0^{60} 0.9x dx = \frac{0.9x^2}{2} \Big|_0^{60} = \frac{0.9(3600)}{2} = 0.45(3600) = 1620 \text{ ft-lb}$

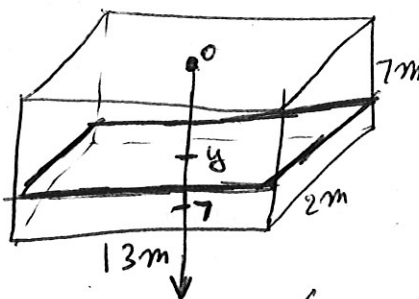


$W_1 = \int_0^{30} 0.9x dx$

$W_2 = \text{weight} \times 30 \text{ ft}$

$= 30(0.9)(30) = \dots$

+



density = 1000 kg/m^3

$g = 9.8 \text{ m/s}^2$

$W = (\text{gravity})(\text{density})(\text{area})(\text{distance}) dy$

$= \int_0^7 (9.8)(1000)(13 \times 2) y dy$

$= 9800(26) \frac{y^2}{2} \Big|_0^7 = \dots$

slice: area = $13 \times 2 \text{ m}^2$

Volume = $13 \times 2 dy \text{ m}^3$

mass = $1000(13 \times 2 dy) \text{ Kg}$

Force = $mg = (9.8)(1000)(13 \times 2 dy) \text{ N}$

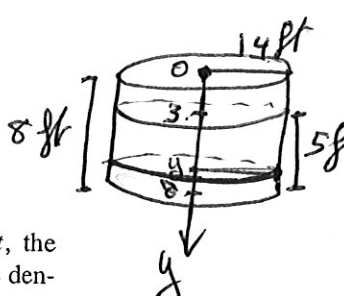
Work to empty a slice = Force \times distance

$= (9.8)(1000)(13 \times 2 dy) y$

$W = \int_0^7 (9.8)(1000)(13 \times 2) y dy$

Book Problem 16

A cylindrical swimming pool has a diameter of 28ft, the sides are 8ft high, and the depth of the water is 5ft. The density of water is 62.5lb/ft³.



distance density = 62.5 lb/ft³

slice:

$$\text{area} = \pi r^2 = \pi (14)^2$$

$$\text{Volume} = \pi (14)^2 dy$$

$$\text{Force} = (62.5) \pi (14)^2 dy$$

$$\text{Work} = (62.5) \pi (14)^2 dy \cdot y$$

$$W = \int_3^8 62.5 \pi (14)^2 y dy$$

$$= 62.5 \pi (14)^2 \left(\frac{y^2}{2} \right) \Big|_3^8 = \dots$$

(a) How much work is required to pump all of the water over the side?

Work = _____ ft-lb.

(b) How much work is required to pump all of the water out of an outlet 3ft above the top?

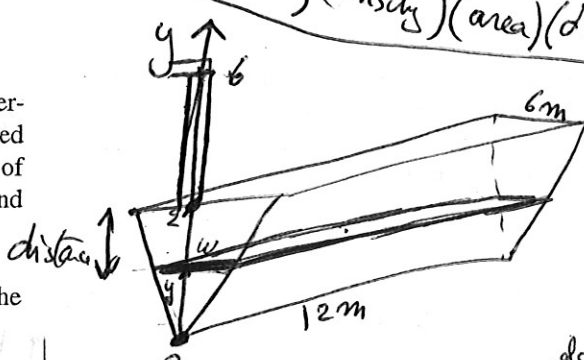
Work = _____ ft-lb.

Book Problem 17

A trough is 12m long, 6m wide, and 2m deep. The vertical cross-section of the trough parallel to an end is shaped like an isosceles triangle (with height 2m, and base, on top, of length 6m). The trough is full of water (density 1000kg/m³ and $g = 9.8\text{m/s}^2$ is the acceleration due to gravity).

a) Find the amount of work in joules required to empty the trough by pumping the water over the top.

b) Find the amount of work in joules required to empty the trough by pumping the water out of a spout 4m high.



distance

W = (density)(area)(distance) dy

slice

$$\frac{w}{6} = \frac{y}{2} \Rightarrow w = 3y$$

a) $w = \int (\text{gravity})(\text{density})(\text{area})(\text{distance}) dy$

Book Problem 18

A hemispherical tank has a radius of 13ft. Given that the density of water is 62.5lb/ft³, find the work required to pump the water out of the tank.

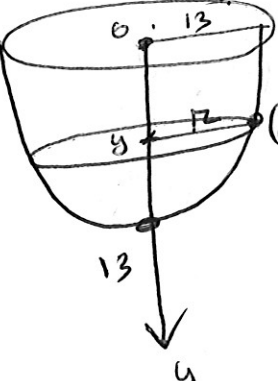
Work = _____

$$W = \int_0^2 (9.8)(1000)(12 \cdot 3y)(2-y) dy$$

$$= 9800(36) \int_0^2 (2y - y^2) dy =$$

$$= (9800)(36) \left(y^2 - \frac{y^3}{3} \right) \Big|_0^2 = \boxed{(9800)(36) \left(\frac{4}{3} \right)} \text{ Joules}$$

b, $W = \int_0^2 (9.8)(1000)(36y)(6-y) dy = \dots$



$$x^2 + y^2 = 13^2 = 169$$

$$r = x = \sqrt{169 - y^2}$$

$$\text{area of slice} = \pi r^2 = \pi (169 - y^2)$$

$$W = \int_0^{13} (62.5) (\pi (169 - y^2)) y dy = 62.5 \pi \int_0^{13} (169y - y^3) dy$$

$$= 62.5 \pi \left(169 \frac{y^2}{2} - \frac{y^4}{4} \right) \Big|_0^{13} = 62.5 \pi \left(169 \cdot \frac{13^2}{2} - \frac{13^4}{4} \right) = \dots$$

Book Problem 21

Newton's Law of Gravitation states that two bodies with masses m_1 and m_2 attract each other with a force

$$F = G \frac{m_1 m_2}{r^2}$$

where r is the distance between the bodies and G is the gravitational constant, whose value is $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$.

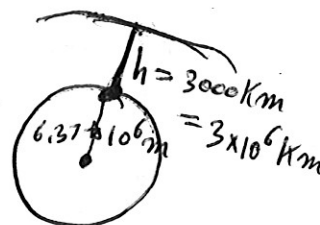
- a) If one 2300 kg body is fixed, find the work needed to move a different body of mass 560 kg from $r = 190 \text{ m}$ to $r = 210 \text{ m}$.

$$\text{Work} = 6.67 \times 10^{-11} \times 2300 \times 560 \left(-\frac{1}{210} + \frac{1}{190} \right) = \dots$$

- b) Compute the work required to launch a 1500-kg satellite vertically to an orbit 3000 km high. You may assume that the Earth's mass is $5.98 \times 10^{24} \text{ kg}$ and is concentrated at its center. Take the radius of the earth to be $6.37 \times 10^6 \text{ m}$.

$$W = \int_{190}^{210} \frac{6.67 \times 10^{-11} \times 2300 \times 560}{r^2} dr = 6.67 \times 10^{-11} \times 2300 \times 560 \left(\frac{1}{190} - \frac{1}{210} \right)$$

$$W = \int_{6.37 \times 10^6}^{9.37 \times 10^6} \frac{6.67 \times 10^{-11} \times 1500 \times 5.98 \times 10^{24}}{r^2} dr$$



Work = _____ J.

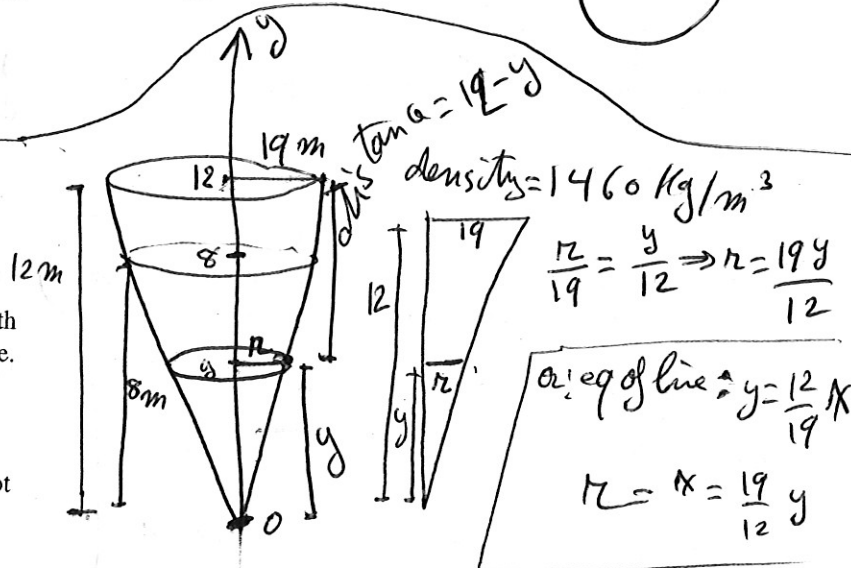
12. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec7.5.extra1.pg

Book Example 4

A tank has the shape of an inverted right circular cone with height 12 m and radius 19 m. It is filled with 8 m of hot chocolate. Assume that the density of the hot chocolate is 1460 kg/m^3 .

Find the work required to empty the tank by pumping the hot chocolate over the top of the tank.

Work = _____ Joules.



$$\begin{aligned} W &= \int_0^8 (\text{gravity}) (\text{density}) (\text{area}) (\text{distance}) dy \\ &= \int_0^8 (9.8) (1460) \left(\pi \left(\frac{19y}{12} \right)^2 \right) (12-y) dy \\ &= (9.8)(1460)\pi \left(\frac{19^2}{12^2} \right) \int_0^8 y^2(12-y) dy \\ &= \int_0^8 (12y^2 - y^3) dy \\ &= \left(\frac{4y^3 - y^4}{4} \right) \Big|_0^8 \end{aligned}$$