WeBWorK assignment number Sec8.2 is due : 11/12/2010 at 10:30pm EST.

When adding the terms of an infinite sequence $\{a_n\}_{n=1}^{\infty}$, we get an infinite **series** $\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots\ldots$

The **nth partial sum** $s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + \ldots\ldots + a_n$ (the sum of the first n terms)

$\sum_{n=1}^{\infty} a_n = s = \lim_{n\to\infty} s_n$

If this limit exists as a real number, then the series is called **convergent** . Otherwise, it is **divergent** .

2 The **Geometric Series** with **common ratio** r , $\sum_{n=1}^{\infty} ar^{n-1} = a + ar + ar^2 + \ldots$ is convergent if $|r| < 1$ and its sum $\boxed{s = \frac{a}{1-r}}$.

The geometric series is divergent if $|r| \geq 1$.

3 The **Harmonic series** $\sum_{n=1}^{\infty} \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$ is **divergent** .

4 The **Telescoping series** $\sum_{n=1}^{\infty} (b_n - b_{n+1})$ is **convergent** if $\lim_{n\to\infty} b_n = L$ and its sum $s = b_1 - L$.

Theorem: If the series $\sum_{n=1}^{\infty} a_n$ is convergent, then $\lim_{n\to\infty} a_n = 0$.

The converse of this theorem is not true.

1 **The Test For Divergence:** If $\lim_{n\to\infty} a_n \neq 0$ or does not exist, then the series $\sum_{n=1}^{\infty} a_n$ is divergent.

Note : if $\lim_{n\to\infty} a_n \neq 0$, then $\sum a_n$ is divergent. But if $\lim_{n\to\infty} a_n = 0$, we know nothing about the convergence or divergence of the series.

---

1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.Extra1.pg

1

Extra Problem

Consider the series $\sum_{n=1}^{\infty} \frac{3}{n+8}$.

$S_3 = \frac{3}{9} + \frac{3}{10} + \frac{3}{11} = \ldots\ldots$

Let $s_n = \sum_{i=1}^{n} \frac{3}{i+8}$ be the n-th partial sum; that is the sum of the first n terms of the series.

$S_6 =$ add 1st six terms

Find s_3 and s_5.

$s_3 =$ ______________

$s_5 =$ ______________

---

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.3.pg

2

Book Problem 3

Determine whether the following series is convergent or divergent. If convergent find the sum, and if divergent enter DIV:

$3 - \frac{3}{4} + \frac{3}{16} - \frac{3}{64} + \ldots =$ 2.4

$\times \frac{-1}{4}$

$\text{sum} = \frac{1\text{st term}}{1-r} = \frac{3}{1 - \frac{-1}{4}} = \frac{3}{\frac{5}{4}} = \frac{12}{5} = 2.4$

geometric $|r| = \left|\frac{-1}{4}\right| < 1$ conv.

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.5.pg

Book Problem 5

(3)

The following series are geometric series. Determine whether each series converges or not.
For the convergent series, enter the sum of the series. For the divergent series, enter DIV .
Also for each series, enter the first term a and the common ratio r.

1st term: $n=1 \Rightarrow 7\left(\frac{3}{6}\right)^{1-1} = 7 = a$

(a) $\sum_{n=1}^{\infty} 7\left(\frac{3}{6}\right)^{n-1} =$ 14, $a =$ 7, and $r =$ 3/6 < 1

$\text{Sum} = \frac{a}{1-r} = \frac{7}{1-\frac{3}{6}} = \frac{7}{1-\frac{1}{2}} = \frac{7}{\frac{1}{2}} = 14$

(b) $\sum_{n=1}^{\infty} \frac{7^n}{6^n} =$ ______, $a = \frac{7}{6}$, and $r = \frac{7}{6} > 1$ Div.

(c) $\sum_{n=0}^{\infty} \frac{3}{10^{2n}} =$ ______, $a =$ 3, and $r = \frac{1}{100}$.

$\sum_{n=0}^{\infty} \frac{3}{10^{2n}} = \sum \frac{3}{(10^2)^n} = \sum \frac{3}{100^n} = \sum_{n=0}^{\infty} 3\left(\frac{1}{100}\right)^n, \ r = \frac{1}{100} < 1$

$a = 3\left(\frac{1}{100}\right)^0 = 3$

Conv, $\text{Sum} = \frac{a}{1-r} = \frac{3}{1-\frac{1}{100}} = \frac{300}{99}$

---

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.7.pg

Book Problem 7

The following series are geometric series. Determine whether each series converges or not.
For the convergent series, enter the sum of the series. For the divergent series, enter DIV .
Also for each series, enter the first term a and the common ratio r.

$\sum \frac{\pi^n}{2^n \cdot 2} = \sum \frac{1}{2}\left(\frac{\pi}{2}\right)^n, \ r = \frac{\pi}{2} = \frac{3.14\ldots}{2} > 1 \Rightarrow$ series Div

(a) $\sum_{n=1}^{\infty} \frac{\pi^n}{2^{n+1}} =$ ______, $a = \frac{\pi}{4}$, and $r = \frac{\pi}{2}$

(b) $\sum_{n=2}^{\infty} \frac{1}{3^{n+1}} =$ ______, $a =$ ___, and $r =$ ___.

$\sum \frac{1}{3^{n+1}} = \sum \frac{1}{3^n \cdot 3} = \sum_{n=2}^{\infty} \frac{1}{3}\left(\frac{1}{3}\right)^n, \ r = \frac{1}{3} < 1$

$a = \frac{1}{3}\left(\frac{1}{3}\right)^2 = \frac{1}{27}$

conv $\text{Sum} = \frac{\frac{1}{27}}{1-\frac{1}{3}} = \frac{1/27}{2/3} = \frac{1}{18}$

(c) $\sum_{n=1}^{\infty} \frac{(-2)^{n-1}}{4^{n-1}} =$ ______, $a =$ ___, and $r =$ ___.

$\sum \frac{(-2)^{n-1}}{4^{n-1}} = \sum \frac{(-2)^n \cdot (-2)^{-1}}{4^n \cdot 4^{-1}} = \sum \frac{(-2)^n \cdot 4}{4^n \cdot (-2)}$

$= \sum -2\left(\frac{-2}{4}\right)^n = \sum_{n=1}^{\infty} -2\left(\frac{-1}{2}\right)^n$

$|r| = \left|-\frac{1}{2}\right| < 1$ Conv series, $\text{sum} = \frac{a}{1-r} = \frac{1}{1-\frac{-1}{2}} = \frac{2}{3}$

---

5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.9.pg

(4)

Book Problem 9

Decide whether the given sequence or series is convergent or divergent. If convergent, enter the limit (for a sequence) or the sum (for a series). If divergent, enter DIV .

(a) The sequence $\left\{\frac{6}{5n}\right\}$. Conv. to 0 $\frac{6}{5}, \frac{6}{10}, \frac{6}{15}, \ldots \text{conv} \ldots 0$, sequence converges to 0

(b) The series $\sum_{n=1}^{\infty}\left(\frac{6}{5n}\right)$. $= \frac{6}{5} \sum \frac{1}{n}$ series Div.

harmonic series

6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.10.pg

Book Problem 10

5

Decide whether the given sequence or series is convergent or divergent. If convergent, enter the limit (for a sequence) or the sum (for a series). If divergent, enter DIV .

(a) The sequence $\left\{\frac{7n}{2n+15}\right\}$. limit $= 7/2 \neq 0$, The sequence is convergent to $\frac{7}{2}$

(b) The series $\sum_{n=1}^{\infty}\left(\frac{7n}{2n+15}\right)$. Div by the Test for Divergence. because $\lim a_n = \frac{7}{2} \neq 0$

---

7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.11.pg

Book Problem 11

Decide whether the given sequence or series is convergent or divergent. If convergent, enter the limit (for a sequence) or the sum (for a series). If divergent, enter DIV .

$\lim \frac{k^8}{k^8-6} = 1 \neq 0$

(a) The sequence $\left\{\frac{k^8}{k^8-6}\right\}$. converges to 1

(b) The series $\sum_{k=1}^{\infty}\left(\frac{k^8}{k^8-6}\right)$. divergent by the test for divergence

---

8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.13.pg

6

Book Problem 13

conv. geo series, $a = \frac{2}{7}$, $r = \frac{2}{7} \Rightarrow \text{Sum} = \frac{\frac{2}{7}}{1-\frac{2}{7}} = \frac{2}{5}$

Determine the sum of the following series:

// , $a = \frac{4}{7}$, $r = \frac{4}{7} \Rightarrow \text{Sum} = \frac{4/7}{1-\frac{4}{7}} = \frac{4}{3}$

$\sum_{n=1}^{\infty}\left(\frac{2^n+4^n}{7^n}\right) = \sum_{1}^{\infty}\left(\frac{2}{7}\right)^n + \sum_{1}^{\infty}\left(\frac{4}{7}\right)^n = \frac{2}{5} + \frac{4}{3} = \frac{6+20}{15} = \frac{26}{15}$

$\sum_{n=1}^{\infty}(0.2^{n-1}-0.7^n) = \sum_{1}^{\infty} 0.2^{n-1} - \sum_{1}^{\infty} 0.7^n = \frac{1}{1-0.2} - \frac{0.7}{1-0.7} = \frac{1}{0.8} - \frac{0.7}{0.3} = \ldots$

---

9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.17.pg

Book Problem 17

Decide whether the given sequence or series is convergent or divergent. If convergent, enter the limit (for a sequence) or the sum (for a series). If divergent, enter DIV .

(a) The sequence $\{\arctan n\}$. $\lim_{n\to\infty} \arctan n = \arctan \infty = \frac{\pi}{2} \Rightarrow$ Convergent sequence

(b) The series $\sum_{n=1}^{\infty}(\arctan n)$. DIV but the series is divergent because $\lim a_n = \frac{\pi}{2} \neq 0$

10. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.19.pg

Book Problem 19

$\frac{4}{n^2-1} = \frac{A(n+1)+B(n-1)}{(n-1)(n+1)} \Rightarrow 4 = A(n+1)+B(n-1)$

if $n=1$: $4 = 2A \Rightarrow A = 2$

if $n=-1$: $4 = -2B \Rightarrow B = -2$

a) Using partial fractions,

$$\frac{4}{n^2-1} = \frac{A}{n-1} + \frac{B}{n+1}, \text{ where } A = \_\_\_ \text{ and } B = \_\_\_.$$

$= \frac{2}{n-1} - \frac{2}{n+1}$

b) Determine whether the following telescoping series is convergent or divergent. If convergent find the sum, and if divergent enter DIV:

$S_n = \underset{n=2}{(2 - \cancel{\tfrac{2}{3}})} + \underset{n=3}{(\tfrac{2}{2} - \cancel{\tfrac{2}{4}})} + \underset{n=4}{(\cancel{\tfrac{2}{3}} - \cancel{\tfrac{2}{5}})} + \underset{n=5}{(\cancel{\tfrac{2}{4}} - \cancel{\tfrac{2}{6}})} + \underset{n=6}{(\cancel{\tfrac{2}{5}} - \cancel{\tfrac{2}{7}})}$

$\cdots + \underset{n-1}{(\cancel{\tfrac{2}{n-2}} - \tfrac{2}{n})} + \underset{n}{(\cancel{\tfrac{2}{n-1}} - \tfrac{2}{n+1})}$

$$\sum_{n=2}^{\infty} \frac{4}{n^2-1} = \underline{\quad 3 \quad}$$

$S_n = 2 + \frac{2}{2} - \frac{2}{n} - \frac{2}{n+1} \xrightarrow{\lim} 2 + \frac{2}{2} = 3$

11. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.21.pg

7

Book Problem 21

$\frac{1}{n(n+1)} = \frac{A}{n} + \frac{B}{n+1} = \frac{A(n+1)+Bn}{n(n+1)} \Rightarrow 1 = A(n+1)+Bn$

if $n=0 \Rightarrow A = 1$

if $n=-1 \Rightarrow B = -1$

$\Big\} \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$

Note that the following series is telescoping. If it is convergent, compute its sum. Otherwise, enter DIV .

= sum of 1st n terms

$$\sum_{n=1}^{\infty} \frac{1}{n(n+1)} = \underline{\boxed{1}}$$

$= \sum_{1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

$S_n = (\tfrac{1}{1} - \cancel{\tfrac{1}{2}}) + (\cancel{\tfrac{1}{2}} - \cancel{\tfrac{1}{3}}) + (\cancel{\tfrac{1}{3}} - \cancel{\tfrac{1}{4}}) + \cdots + (\cancel{\tfrac{1}{n}} - \tfrac{1}{n+1}) = 1 - \frac{1}{n+1}$

$S = \lim_{n\to\infty} S_n = 1 - \frac{1}{\infty} = 1 - 0 = 1$

12. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.22.pg

8

Book Problem 22

Determine whether the following telescoping series are convergent or divergent. If convergent, enter the sum. If not, enter DIV.

$S_n = (\ln 1 - \cancel{\ln 2}) + (\cancel{\ln 2} - \cancel{\ln 3}) + \cdots + (\cancel{\ln n} - \ln(n+1))$

$$\sum_{n=1}^{\infty} \ln\frac{n}{n+1} = \sum_{n=1}^{\infty} \ln n - \ln(n+1) = -\infty \quad \text{DIV}$$

$S_n = \underset{0}{\ln 1} - \ln(n+1) = -\ln(n+1)$

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} -\ln(n+1) = -\infty$

$$\sum_{n=1}^{\infty} (e^{-6n} - e^{-6(n+1)}) = \underline{\quad e^{-6} \quad}$$

$\hookrightarrow S_n = (e^{-6} - \cancel{e^{-12}}) + (\cancel{e^{-12}} - \cancel{e^{-18}}) + (\cancel{e^{-18}} - \cancel{e^{-24}}) + \cdots + (\cancel{e^{-6n}} - e^{-6(n+1)})$

$S_n = e^{-6} - e^{-6(n+1)}$

$S = \lim_{n\to\infty} S_n = \lim_{n\to\infty} (e^{-6} - e^{-6(n+1)}) = e^{-6} - e^{-\infty} = e^{-6} - 0 = e^{-6}$

$4.909090\ldots = 4 + .90 + .0090 + .000090 + \ldots$

$\times 0.01 \quad \times 0.01$

geometric $r < 1$

13. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.23.pg

Book Problem 23

$= 4 + \frac{a}{1-r} = 4 + \frac{.90}{1 - 0.01} = 4 + \frac{.90}{.99}$

$= 4 + \frac{90}{99} = 4 + \frac{10}{11} = \frac{54}{11}$

Express 4.90909... as a rational number, in the form $\frac{p}{q}$ where p and q are positive integers with no common factors.

$p =$ 54 and $q =$ 11

14. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.25.pg

Book Problem 25

$3.684684\ldots = 3 + .684 + .000684 + .000000684 + \ldots$

$= 3 + .684 + .684(.001) + .684(.001)^2 + \ldots$

geometric $r = .001$

Express 3.684684684... as a rational number, in the form $\frac{p}{q}$ where p and q have no common factors.

$= 3 + \frac{.684}{1 - .001} = 3 + \frac{684}{999} = 3 + \frac{76}{111} = \frac{409}{111}$

$p =$ 409 and $q =$ 111

15. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.27.pg

Book Problem 27

a) Find the values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{6^n}$ converges. $= \sum \left(\frac{x}{6}\right)^n$ convergent geometric if

Answer: $-6 < x < 6$

$|r| = \left|\frac{x}{6}\right| < 1 \Rightarrow |x| < 6$

$\Rightarrow -6 < x < 6$

(b) Find the sum of the series for those values of x .

$S = \frac{a}{1-r} = \frac{\frac{x}{6}}{1 - \frac{x}{6}} = \frac{x}{6-x}$

Sum = $\frac{x}{6-x}$. (your answer should be a function of x)

16. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.2.31.pg

Book Problem 31

If the nth partial sum of a series $\sum_{n=1}^{\infty} a_n$ is $s_n = \frac{n-2}{n+6}$, then

$a_n =$ ______ and $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{n-2}{n+6} = 1$

$S_n = a_1 + a_2 + \cdots + a_{n-1} + a_n$

$S_{n-1} = a_1 + a_2 + \cdots + a_{n-1}$ $-$

$S_n - S_{n-1} = a_n \Rightarrow a_n = \frac{n-2}{n+6} - \frac{(n-1)-2}{(n-1)+6} = \boxed{\frac{n-2}{n+6} - \frac{n-3}{n+5}}$

$\frac{76}{}$ $3 + \frac{684}{999}$ $\frac{}{111}$ $\frac{409}{111}$