finish it

WeBWorK assignment number Sec8.3 is due : 11/18/2010 at 10:30pm EST.

The Test For Divergence: If $\lim_{n\to\infty} a_n \neq 0$ or does not exist, then the series $\sum a_n$ is divergent.

The **Geometric Series** $\sum ar^{n-1}$ is convergent if $|r| < 1$ and its sum $s = \frac{a}{1-r}$. This series is divergent if $|r| \geq 1$.

The **Harmonic series** $\sum \frac{1}{n} = 1 + \frac{1}{2} + \frac{1}{3} + \ldots$ is **divergent** .

The **Telescoping series** $\sum_{n=1}^{\infty} (b_n - b_{n+1})$ is **convergent** if $\lim_{n\to\infty} b_n = L$ and its sum $s = b_1 - L$.

The **p-series** $\sum \frac{1}{n^p}$ is convergent if $p > 1$ and divergent if $p \leq 1$.

The Integral Test: Suppose f is continuous, positive, decreasing function on $[1,\infty)$ and $a_n = f(n)$. Then the series $\sum a_n$ is convergent if and only if $\int_1^\infty f(x)dx$ is convergent.

The Comparison Test: Suppose both series have positive terms,
a) If $\sum b_n$ is convergent and $a_n \leq b_n$, then $\sum a_n$ is also convergent.
b) If $\sum b_n$ is divergent and $a_n \geq b_n$, then $\sum a_n$ is also divergent.

The Limit Comparison Test: Suppose $\sum a_n$ and $\sum b_n$ have positive terms. If $\lim_{n\to\infty} \frac{a_n}{b_n} = c > 0$ and finite number, then either both series converge or both diverge.

Note that the Integral and Comparison tests apply only to series with **positive** terms.

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1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.7.pg

Book Problem 7

A) Find $\int_2^\infty \frac{dx}{(5x-2)^6} =$ $\frac{1}{25(8^5)}$.(Enter its value if convergent and DIV if divergent)

$$\int_2^\infty = \lim_{t\to\infty} \int_2^t = \lim_{t\to\infty} \left.\frac{(5x-2)^{-5}}{-5(5)}\right|_2^t$$

$$= \lim \left(\underbrace{\frac{-1}{25(5t-2)^5}}_{\to 0} - \frac{-1}{25(10-2)^5} \right) = \frac{1}{25(8)^5}$$

B) Use the integral test to determine whether $\sum_{n=2}^{\infty} \frac{1}{(5n-2)^6}$ is convergent or divergent?
Enter CONV if the series is convergent and DIV if divergent.

— Conv.

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2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.8.pg

Book Problem 8

A) Find $\int_1^\infty \frac{2dx}{x^2+1} =$ $\pi/2$.(Enter its value if convergent and DIV if divergent)

$$\int_1^\infty = \lim_{t\to\infty} \int_1^t = \lim_{t\to\infty} 2\arctan x\Big|_1^t$$

$$= \lim_{t\to\infty} (2\arctan t - 2\arctan 1)$$

$$= 2\arctan\infty - 2\arctan 1$$

$$= 2\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{4}\right) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$$

B) Determine whether $\sum_{n=1}^{\infty} \frac{2}{n^2+1}$ is convergent or divergent.

Enter CONV if the series is convergent and DIV if divergent.

— Conv.

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3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.9.pg

Book Problem 9

Each of the following statements is an attempt to show that a given series is convergent or divergent. For each statement, enter C (for "correct") if the argument is valid, or enter I (for "incorrect") if any part of the argument is flawed. (Note: if the conclusion is true but the argument that led to it was wrong, you must enter I.)

C 1. For all $n > 1$, $\frac{1}{n^2+n+6} < \frac{1}{n^2}$, and the series $\sum \frac{1}{n^2}$ converges.

So by the Comparison Test, the series $\sum \frac{1}{n^2+n+6}$ converges.

I 2. For all $n > 3$, $\cancel{\frac{1}{n^2-6} < \frac{1}{n^2}}$, and the series $\sum \frac{1}{n^2}$ converges.
So by the Comparison Test, the series $\sum \frac{1}{n^2-6}$ converges.

C 3. For all $n > 1$, $\frac{\sqrt{n+1}}{n} > \frac{1}{n}$, and the series $\sum \frac{1}{n}$ diverges.
So by the Comparison Test, the series $\sum \frac{\sqrt{n+1}}{n}$ diverges.

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4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.11.pg

Book Problem 11

Select the FIRST correct answer:

A. Divergent harmonic series
B. Divergent geometric series
C. Convergent geometric series
D. Divergent p-series
E. Convergent p-series

A 1. $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ $= \Sigma \frac{1}{n}$

E 2. $1+\frac{1}{4}+\frac{1}{9}+\frac{1}{16}+\ldots$ $= \Sigma \frac{1}{n^2}$

C 3. $1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$ $= \Sigma \frac{1}{3^n}$

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5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.12.pg

Book Problem 12

Enter CONV if the given series converges or DIV if it diverges.

P-series
Conv if $P>1$
Div if $P\le 1$

Div 1. $\sum_{n=1}^{\infty} \frac{3}{n}$ $P=1\le 1$

Div 2. $\sum_{n=1}^{\infty} \frac{6}{n^{-4}}$ $P=-4\le 1$

Conv 3. $\sum_{n=1}^{\infty} \frac{2}{n\sqrt{n}}$ $P=1.5=\frac{3}{2}>1$

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6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.13.pg

Book Problem 13

let $u=-x^3 \Rightarrow du=-3x^2dx \Rightarrow dx=\frac{du}{-3x^2}$

A) Find the value of $\int_1^{\infty} 7x^2e^{-x^3}\,dx = \int 7x^2 e^u \frac{du}{-3x^2} = \frac{-7}{3}\int e^u du = -\frac{7}{3}e^u\Big/ = -\frac{7}{3}e^{-x^3}\Big/_1^{\infty} = \lim_{t\to\infty}$

$= -\frac{7}{3}\left(e^{-\infty} - e^{-1}\right) = \boxed{\frac{7}{3e}}$

$e^{-\infty} \to 0$

B) Determine whether $\sum_{n=1}^{\infty} 7n^2e^{-n^3}$ is convergent or divergent

Enter CONV if series is convergent, DIV if series is divergent.

Conv

The integral is convergent
$\Rightarrow$ the series is also convergent by the integral test

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7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.15.pg

Book Problem 15

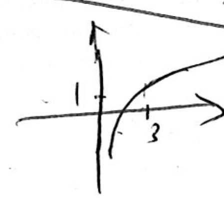
Select the FIRST correct reason why the given series diverges.

A. Divergent p-series
B. Divergent geometric series
C. Comparison with a divergent p-series
D. Diverges because the terms don't have limit zero
E. Integral test

$\Sigma \frac{1}{n}$ Div.

$\frac{\ln n}{n} > \frac{1}{n}$

$\Sigma \frac{\ln n}{n} > \Sigma \frac{1}{n}$ Div



C 1. $\sum_{n=3}^{\infty} \frac{\ln(n)}{n}$

D 2. $\sum_{n=3}^{\infty} \ln(n)$, $\lim_{n\to\infty} \ln n = \ln\infty \ne 0$

A 3. $\sum_{n=3}^{\infty} \frac{1}{n}$

E 4. $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$

$\frac{1}{n\ln(n)} < \frac{1}{n}$

$\Sigma \frac{1}{n\ln n} < \Sigma \frac{1}{n}$ Div

The comparison test doesn't work

let $u=\ln x$
$du=\frac{1}{x}dx$

$\int_3^{\infty} \frac{1}{x\ln x}dx = \int \frac{1}{u}du = \ln u = \ln(\ln x)\Big/_1^{\infty} = \ln(\ln\infty) - \ldots = \infty$ Div

8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.16.pg

Book Problem 16

Use the limit comparison test with a p-series to determine whether the following series are convergent or divergent. Enter CONV for convergent, DIV for divergent , and the value of p.

A) $\sum \frac{n^3-1}{7n^8+1}$ is Conv with $p =$ 5 compare with $\Sigma \frac{1}{n^5}$

B) $\sum \frac{n^4}{n^5+3}$ is Div with $p =$ 1 compare with $\Sigma \frac{1}{n}$

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9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.17.pg

Book Problem 17

Select the FIRST correct answer:

A. Convergent by comparison with a geometric series
B. Convergent by comparison with a p-series
C. Convergent by the integral test
D. Divergent by comparison with a geometric series
E. Divergent by comparison with a p-series

B 1. $\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^7+8}$

A 2. $\sum_{n=1}^{\infty} \frac{1+\sin n}{3^n}$

B 3. $\sum_{n=1}^{\infty} \frac{\sin^2(3n)}{n^2}$

1. $\frac{\cos^2 n}{n^7+8} \leq \frac{1}{n^7+8} < \frac{1}{n^7}$

$\Sigma \frac{1}{n^7}$ conv p-series $p=7>1$

2. $\frac{1+\sin n}{3^n} \leq \frac{2}{3^n}$, $\Sigma \frac{1}{3^n}$ conv. geo.

3. $\frac{\sin^2(3n)}{n^2} \leq \frac{1}{n^2}$, $\Sigma \frac{1}{n^2}$ conv, ~~geo~~ p-series $p=2>1$

10. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.19.pg

Book Problem 19

Select the FIRST correct answer:

A. Convergent by comparison with a geometric series
B. Convergent by limit comparison with a geometric series
C. Divergent by comparison with a geometric series
D. Divergent by limit comparison with a geometric series

B 1. $\sum_{n=1}^{\infty} \frac{1}{5^n - 4}$

A 2. $\sum_{n=1}^{\infty} \frac{n-1}{n5^n}$

C 3. $\sum_{n=1}^{\infty} \frac{6+9^n}{4^n}$

1. $\frac{1}{5^n-4} > \frac{1}{5^n}$, $\sum \frac{1}{5^n}$ is conv. geo

so no conclusion with comparison

Do limit comparison test

$\lim \frac{a_n}{b_n} = \lim \frac{\frac{1}{5^n-4}}{\frac{1}{5^n}} = \lim \frac{5^n}{5^n-4} = 1 > 0$

$\Rightarrow$ both series be have the same

2. $\frac{n-1}{n5^n} < \frac{n}{n5^n} = \frac{1}{5^n}$

so $\sum \frac{n-1}{n5^n}$ is conv by comparison wi $\sum \frac{1}{5^n}$ conv. geo.

3. $\frac{6+9^n}{4^n} > \frac{9^n}{4^n} = \left(\frac{9}{4}\right)^n$

$\sum \left(\frac{9}{4}\right)^n$ is div geo. $r = \frac{9}{4} > 1 \Rightarrow \sum \frac{6+9^n}{4^n}$ is also div

11. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.21.pg

Book Problem 21

Consider the three series

$$A_n = \frac{1}{n}, \quad B_n = \frac{1}{\sqrt{n}}, \quad C_n = \frac{1}{n^{3/2}}.$$

For each of the series below, enter the letter (A,B, or C) of the series above that it can be compared to with the Limit Comparison Test.

C 1. $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n^2+4}$

B 2. $\sum_{n=1}^{\infty} \frac{1}{6+\sqrt{n}}$

A 3. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n(n+1)}} = \sum \frac{1}{\sqrt{n^2+n}}$ compare with $\sum \frac{1}{\sqrt{n^2}} = \sum \frac{1}{n}$

A 4. $\sum_{n=1}^{\infty} \frac{1}{5n+1}$

12. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.23.pg

Book Problem 23

Select the FIRST correct answer:

A. Convergent by comparison with a p-series
B. Convergent by limit comparison with a p-series
C. Divergent by comparison with a p-series
D. Divergent by limit comparison with a p-series

A 1. $\sum_{n=1}^{\infty} \frac{7+(-1)^n}{n\sqrt{n}}$

B 2. $\sum_{n=1}^{\infty} \frac{n-2}{\sqrt{n^9-1}}$

D 3. $\sum_{n=1}^{\infty} \frac{4}{5n+1}$ $\quad \lim \frac{\frac{4}{5n+1}}{\frac{1}{n}} = \lim \frac{4n}{5n+1} = \frac{4}{5} > 0 \Rightarrow$ both series behave the same

$\sum \frac{1}{n}$ Div $\Rightarrow$ Div

1. $\frac{7+(-1)^n}{n\sqrt{n}} \le \frac{7}{n\sqrt{n}}$, $\sum \frac{1}{n\sqrt{n}} = \sum \frac{1}{n^{3/2}}$ conv. p-series $p = 3/2 > 1$

2. $\frac{n-2}{\sqrt{n^9-1}} < \frac{n}{\sqrt{n^9-1}}$ cannot continue with comparison

limit comparison: $\lim \frac{a_n}{b_n} = \lim \frac{\frac{n-2}{\sqrt{n^9-1}}}{\frac{1}{n^{3.5}}}$

$= \lim \frac{(n-2)n^{3.5}}{\sqrt{n^9-1}} = 1 > 0$

both series behave the same and $\sum \frac{1}{n^{3.5}}$ conv.

13. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.3.Extra1.pg

Extra Problem

Test each of the following series for convergence by either the Comparison Test or the Limit Comparison Test. Enter CONV if it converges or DIV if it diverges.

Conv 1. $\sum_{n=1}^{\infty} \frac{\cos^2(n)\sqrt{n}}{n^4}$ $\quad \le \sum \frac{\sqrt{n}}{n^4} = \sum \frac{1}{n^{3.5}}$ conv. p-series

Conv 2. $\sum_{n=1}^{\infty} \frac{3n^6 - n^5 + 8\sqrt{n}}{4n^8 - n^4 + 2}$ $\quad$ limit comparison with $\sum \frac{1}{n^2}$ conv

Conv 3. $\sum_{n=1}^{\infty} \frac{8n^4}{n^6+9}$ $\quad \longrightarrow$ 〃 〃 〃 〃 〃

Div 4. $\sum_{n=1}^{\infty} \frac{(\ln(n))^2}{n+4}$ $\quad > \sum \frac{1}{n+4}$ Div by limit comparison with harmonic

Div 5. $\sum_{n=1}^{\infty} \frac{8n^4}{n^5+9}$ $\quad \to$ limit comparison with $\sum \frac{1}{n}$ Div

1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.4.5.pg

Book Problem 5

Match each of the following series with the correct statement:

A. The series is absolutely convergent.
C. The series is conditionally convergent.
D. The series diverges.

C 1. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}}$ conv. alt series because the terms are ↓ with $\lim b_n = \lim \frac{1}{\sqrt{n}} = 0$ → $\sum \left| \frac{(-1)^{n-1}}{\sqrt{n}} \right| = \sum \frac{1}{\sqrt{n}}$ Div p-series $p = .5 \le 1$

C 2. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$

A 3. $\sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}$, $\sum \left| \frac{(-1)^n}{n\sqrt{n}} \right| = \sum \frac{1}{n^{1.5}}$ conv p-series, $p = 1.5 > 1$

D 4. $\sum_{n=1}^{\infty} (-1)^n \frac{3n}{3n-1}$ → $\lim b_n = \lim \frac{3n}{3n-1} = 1 \Rightarrow$ Series is Div by the test for Divergence

A 5. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^5}$, $\sum \left| \frac{(-1)^{n-1}}{n^5} \right| = \sum \frac{1}{n^5}$ conv p-series $p = 5 > 1$

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2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.4.7.pg

Book Problem 7

$\cos(n\pi) = (-1)^n$ because $\cos(0\pi) = 1$
$\cos(1\pi) = -1$
$\cos(2\pi) = 1$
⋮

Match each of the following series with the correct statement:

A. The series is convergent by the Alternating Series Test.
B. The series is divergent by the Test for Divergence.

A 1. $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^4} = \sum \frac{(-1)^n}{n^4}$ conv. by alt series test because terms $b_n = \frac{1}{n^4}$ are decreasing with limit $= 0$

B 2. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{\ln n}$, $\lim \frac{x}{\ln x} = \lim \frac{1}{1/x} = \lim x = \infty \Rightarrow$ Series Div by the test for Div.

A 3. $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$

B 4. $\sum_{n=1}^{\infty} (-1)^n \frac{4n+3}{6n-1}$, $\lim \frac{4n+3}{6n-1} = \frac{4}{6} \ne 0 \Rightarrow$ //

A 5. $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}$, $\lim \frac{\ln x}{x} = \lim \frac{1/x}{1} = 0$ and the terms are decreasing (find $y' < 0$) so alt. series test ⇒ Conv.

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3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.4.9.pg

Book Problem 9

How many terms of the series do we need to add in order to find the sum to the indicated accuracy?

$$\sum_{n=1}^{\infty} \frac{(-0.55)^n}{n!}, \quad \text{error} \le 0.0005.$$

$\sum_{n=1}^{\infty} \frac{(-0.55)^n}{n!} = \frac{-0.55}{1!} + \frac{0.55^2}{2!} - \frac{0.55^3}{3!} + \frac{0.55^4}{4!} - \cdots$

$= -0.55 + .15125 - .027729 + .0038 - .000419$ ~~.000038~~

if we add only 4 terms the error is $< .000419$ acceptable

Answer: 4

Note: Enter the smallest possible integer.
