

Book Problem 5

Match each of the following series with the correct statement:

- A. The series is absolutely convergent.
 C. The series is conditionally convergent.
 D. The series diverges.

$$C_1. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{\sqrt{n}} \xrightarrow{\text{comp. alt series}} \sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{\sqrt{n}} \text{ Div p-series } p=0.5 \leq 1$$
because the terms are \downarrow with $\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} = 0$

$$C_2. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n}$$

$$A_3. \sum_{n=1}^{\infty} \frac{(-1)^n}{n\sqrt{n}}, \sum_{n=1}^{\infty} \left| \frac{(-1)^n}{n\sqrt{n}} \right| = \sum_{n=1}^{\infty} \frac{1}{n^{1.5}} \text{ conv p-series, } p=1.5 > 1$$

$$D_4. \sum_{n=1}^{\infty} (-1)^n \frac{3n}{3n-1} \xrightarrow{\lim b_n = \lim \frac{3n}{3n-1} = 1} \text{ series is Div by the test for Divergence}$$

$$A_5. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^5}, \sum_{n=1}^{\infty} \left| \frac{(-1)^{n-1}}{n^5} \right| = \sum_{n=1}^{\infty} \frac{1}{n^5} \text{ conv p-series } p=5 > 1$$

Book Problem 7

$$\cos(n\pi) = (-1)^n \text{ because } \begin{aligned} \cos(0\pi) &= 1 \\ \cos(1\pi) &= -1 \\ \cos(2\pi) &= 1 \\ &\vdots \end{aligned}$$

Match each of the following series with the correct statement:

- A. The series is convergent by the Alternating Series Test.
 B. The series is divergent by the Test for Divergence.

$$A_1. \sum_{n=1}^{\infty} \frac{\cos n\pi}{n^4} = \sum_{n=1}^{\infty} \frac{(-1)^n}{n^4} \text{ conv. by alt series test because terms } b_n = \frac{1}{n^4} \text{ are decreasing with limit } = 0$$

$$B_2. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{\ln n}, \lim_{n \rightarrow \infty} \frac{n}{\ln n} = \lim_{n \rightarrow \infty} \frac{1}{1/n} = \lim_{n \rightarrow \infty} n = \infty \Rightarrow \text{series Div by the test for Div.}$$

$$A_3. \sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n^3}$$

$$B_4. \sum_{n=1}^{\infty} (-1)^n \frac{4n+3}{6n-1}, \lim_{n \rightarrow \infty} \frac{4n+3}{6n-1} = \frac{4}{6} \neq 0 \Rightarrow //$$

$$A_5. \sum_{n=1}^{\infty} (-1)^{n-1} \frac{\ln n}{n}, \lim_{n \rightarrow \infty} \frac{\ln n}{n} = \lim_{n \rightarrow \infty} \frac{1/n}{1} = 0 \text{ and the terms are decreasing (find } y' < 0) \text{ so alt. series test } \Rightarrow \text{conv.}$$

Book Problem 9

How many terms of the series do we need to add in order to find the sum to the indicated accuracy?

$$\sum_{n=1}^{\infty} \frac{(-0.55)^n}{n!}, \text{ error} \leq 0.0005.$$

Answer: 4

Note: Enter the smallest possible integer.

$$\sum_{n=1}^{\infty} \frac{(-0.55)^n}{n!} = \frac{-0.55}{1!} + \frac{0.55^2}{2!} - \frac{0.55^3}{3!} + \frac{0.55^4}{4!} - \frac{0.55^5}{5!} + \dots$$

$$= -0.55 + 0.15125 - 0.027729 + 0.0038 - 0.000419 + \dots$$

if we add only 4 terms the error is < 0.000419 acceptable

Book Problem 20

Consider the series $\sum_{n=1}^{\infty} (-1)^n \frac{n^7}{6^n}$.

Attempt the Ratio Test to determine whether the series converges.

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{(n+1)^7}{n^7 \cdot 6}, L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \frac{1}{6}$$

Enter the numerical value of the limit L if it converges, INF if it diverges to infinity, or DIV if it diverges but not to infinity.

Which of the following statements is true?

- A. The Ratio Test says that the series converges absolutely.
 B. The Ratio Test says that the series diverges.
 C. The Ratio Test says that the series converges conditionally.
 D. The Ratio Test is inconclusive, but the series converges absolutely by another test or tests.
 E. The Ratio Test is inconclusive, but the series diverges by another test or tests.
 F. The Ratio Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here: ☐ A

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{(-1)^{n+1} \frac{(n+1)^7}{6^{n+1}}}{(-1)^n \frac{n^7}{6^n}} \right| = \lim_{n \rightarrow \infty} \left| \frac{(n+1)^7}{6^{n+1}} \cdot \frac{6^n}{n^7} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{(n+1)^7}{n^7} \cdot \frac{6^n}{6^{n+1}} \right| = \lim_{n \rightarrow \infty} \left| \left(\frac{n+1}{n} \right)^7 \cdot \frac{1}{6} \right| \\ &= \frac{1}{6} < 1 \end{aligned}$$

so the series is abs. conv.

so it is also convergent.

Book Problem 21

Consider the series $\sum_{n=1}^{\infty} \frac{(-5)^n}{n!}$.

Attempt the Ratio Test to determine whether the series converges.

$$\left| \frac{a_{n+1}}{a_n} \right| = \frac{5}{n+1}, L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = 0$$

Enter the numerical value of the limit L if it converges, INF if it diverges to infinity, or DIV if it diverges but not to infinity.

Which of the following statements is true?

- A. The Ratio Test says that the series converges absolutely.
 B. The Ratio Test says that the series diverges.
 C. The Ratio Test says that the series converges conditionally.
 D. The Ratio Test is inconclusive, but the series converges absolutely by another test or tests.
 E. The Ratio Test is inconclusive, but the series diverges by another test or tests.
 F. The Ratio Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here: ☐ A

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{(-5)^{n+1}}{(n+1)!} \cdot \frac{n!}{(-5)^n} \right| = \lim_{n \rightarrow \infty} \left| \frac{5^{n+1}}{(n+1)!} \cdot \frac{n!}{5^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{5^{n+1}}{5^n} \cdot \frac{n!}{(n+1)!} \right| \\ &= \lim_{n \rightarrow \infty} \left| 5 \cdot \left(\frac{1}{n+1} \right) \right| = 0 < 1 \end{aligned}$$

$\frac{1}{\infty} = 0$

so the series is abs. convergent

$$\begin{aligned} n! &= n(n-1)(n-2)\dots\dots\dots 1 \\ (n+1)! &= (n+1)n(n-1)(n-2)\dots\dots\dots n+1 \\ \frac{5!}{6!} &= \frac{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1} = \frac{1}{6} \end{aligned}$$

Book Problem 25

Consider the series $\sum_{n=1}^{\infty} \frac{(-9)^n}{(7n^2+6)7^{n+3}}$.

Use the Ratio Test to decide whether the series converges.

$$L = \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \underline{9/7}$$

Enter the numerical value of the limit L if it converges, INF if it diverges to infinity, or DIV if it diverges but not to infinity.

$$\begin{aligned} \lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| &= \lim_{n \rightarrow \infty} \left| \frac{\frac{(-9)^{n+1}}{(7(n+1)^2+6)7^{n+4}}}{\frac{(-9)^n}{(7n^2+6)7^{n+3}}} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{9^{n+1}}{[7(n+1)^2+6]7^{n+4}} \cdot \frac{(7n^2+6)7^{n+3}}{9^n} \right| \\ &= \lim_{n \rightarrow \infty} \left| \frac{9^{n+1}}{9^n} \cdot \frac{7^{n+3}}{7^{n+4}} \cdot \frac{(7n^2+6)}{(7(n+1)^2+6)} \right| \\ &= \lim_{n \rightarrow \infty} \left| 9 \cdot \frac{1}{7} \cdot \frac{7n^2+6}{7(n+1)^2+6} \right| = \frac{9}{7} > 1 \end{aligned}$$

Series is Div.

Which of the following statements is true?

- A. The Ratio Test says that the series converges absolutely.
 B. The Ratio Test says that the series diverges.
 C. The Ratio Test says that the series converges conditionally.
 D. The Ratio Test is inconclusive, but the series converges absolutely by another test or tests.
 E. The Ratio Test is inconclusive, but the series diverges by another test or tests.
 F. The Ratio Test is inconclusive, but the series converges conditionally by another test or tests.

Enter the letter for your choice here: ☐ ? ☒ B

Book Problem 27

$$|\sin x| \leq 1$$

$$|\cos x| \leq 1$$

Match each of the following series with the first correct statement.

- A. The series is absolutely convergent using comparison with a p-series.
 B. The series is absolutely convergent using comparison with a geometric series.
 C. The series is absolutely convergent using the Ratio Test.
 D. The series diverges.

① $\sum_{n=1}^{\infty} \frac{\sin 7n}{6^n}$, $\left| \frac{\sin 7n}{6^n} \right| \leq \frac{1}{6^n}$, $\sum \frac{1}{6^n}$ is a geo, $r = \frac{1}{6} < 1$ Conv.

② $\sum_{n=1}^{\infty} \frac{\cos 4n}{n!}$ → ratio test, $\lim_{n \rightarrow \infty} \left| \frac{a_{n+1}}{a_n} \right| = \lim_{n \rightarrow \infty} \left| \frac{\frac{\cos 4(n+1)}{(n+1)!}}{\frac{\cos 4n}{n!}} \right| = \lim_{n \rightarrow \infty} \left| \frac{\cos 4(n+1)}{\cos 4n} \cdot \frac{n!}{(n+1)!} \right|$
 $= \lim_{n \rightarrow \infty} \left| 1 \cdot \frac{1}{n+1} \right| = 0 < 1$

③ $\sum_{n=1}^{\infty} \frac{(-1)^n \arctan n}{n^3}$

④ $\sum_{n=1}^{\infty} \frac{\sin 5n}{n^3}$

$\left| \frac{\sin 5n}{n^3} \right| < \frac{1}{n^3}$, $\sum \frac{1}{n^3}$ is a conv. p-series

Book Problem 31

Use either the Ratio Test or the Root Test to decide whether the following series converge:

- A. The series is absolutely convergent.
 B. The series is conditionally convergent.
 C. The series is divergent.

1. $\sum_{n=1}^{\infty} \frac{n^n}{3^{1+4n}}$ _____

2. $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{n^n}$ _____

Book Problem 39

For each series, determine whether the Ratio Test is conclusive or not.

Enter C (for "Conclusive") or enter I (for "Inconclusive").

C 1. $\sum \frac{n}{5^n}$ $\lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \left| \frac{\frac{n+1}{5^{n+1}}}{\frac{n}{5^n}} \right| = \lim \left| \frac{n+1}{5^{n+1}} \cdot \frac{5^n}{n} \right| = \lim \left| \frac{n+1}{n} \cdot \frac{1}{5} \right| = \frac{1}{5} < 1$

2. $\sum \frac{(-4)^{n-1}}{\sqrt{n}}$

3. $\sum \frac{\sqrt{n}}{n^2 + 4}$

I 4. $\sum \frac{1}{n^5}$

$\lim \left| \frac{a_{n+1}}{a_n} \right|$

$\lim \left| \frac{a_{n+1}}{a_n} \right| = \lim \left| \frac{\frac{1}{(n+1)^5}}{\frac{1}{n^5}} \right| = \lim \left| \frac{n^5}{(n+1)^5} \right| = 1$

Ratio Test
is inconclusive