WeBWorK assignment number Sec8.5 is due : 11/23/2010 at 11:30pm EST.

Remember: always check the endpoints

$\frac{x^{n+1}}{x^n} = \frac{x^n \cdot x}{x^n} = x$

For any **power series** $\sum_{n=0}^{\infty} c_n(x-a)^n$ there are only three cases:

1) The series converges only when $x = a$.
2) The series converges for all x.
3) There is a positive number R such that the series converges for $|x-a| < R$ and diverges for $|x-a| > R$

The number R is called the **Radius of Convergence** of the power series.
The interval over which the series converges is called the **Interval of Convergence** of the power series.
There are four possibilities for the interval of convergence: $(a-R, a+R), (a-R, a+R], [a-R, a+R), [a-R, a+R]$.

The **Ratio Test** and the **Root Test** can be used to determine the radius of convergence R.
The endpoints must be checked with some other test.

1. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.3.pg

Book Problem 3

Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(x)^n}{\sqrt[6]{n}}$.

The series is convergent from $x =$ -1, left end included (enter Y or N): Y

to $x =$ 1, right end included (enter Y or N): N

The radius of convergence is R = 1.

R=1

-1 0 1

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{x^{n+1}}{\sqrt[6]{n+1}}}{\frac{x^n}{\sqrt[6]{n}}}\right| = \left|\frac{x^{n+1}}{\sqrt[6]{n+1}} \cdot \frac{\sqrt[6]{n}}{x^n}\right| = \left|x \cdot \frac{\sqrt[6]{n}}{\sqrt[6]{n+1}}\right|$

$\lim \left|\frac{a_{n+1}}{a_n}\right| = |x| < 1 \xrightarrow{R} \boxed{-1 \leq x < 1}$

if $x = 1$: $\sum \frac{1^n}{\sqrt[6]{n}} = \sum \frac{1}{n^{1/6}}$ is div p-series $p = \frac{1}{6} \leq 1$

if $x = -1$: $\sum \frac{(-1)^n}{\sqrt[6]{n}}$ is a conv. alt series because $\lim b_n = \lim \frac{1}{\sqrt[6]{n}} = 0$ & $b_n \searrow$ to 0.

2. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.4.pg

Book Problem 4

Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^n(x^n)}{(n+11)}$.

The series is convergent from $x =$ ___, left end included (enter Y or N): ___

to $x =$ ___, right end included (enter Y or N): ___

The radius of convergence is R = ___

3. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.5.pg

Book Problem 5

Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}x^n}{n^9}$.

The series is convergent from $x =$ ___, left end included (enter Y or N): ___

to $x =$ ___, right end included (enter Y or N): ___

The radius of convergence is R = ___.

4. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.6.pg

Book Problem 6

Find the interval of convergence of the power series $\sum_{n=1}^{\infty} \sqrt[9]{n}x^n$.

The series is convergent from $x =$ ___, left end included (enter Y or N): ___

to $x =$ ___, right end included (enter Y or N): ___

The radius of convergence is R = ___.

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5. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.7.pg

Book Problem 7

$\lim\left|\frac{a_{n+1}}{a_n}\right| = \lim\left|\frac{(-7x)^{n+1}}{(n+1)!}\cdot\frac{n!}{(-7x)}\right| = \lim\left|7x\cdot\frac{n!}{(n+1)!}\right| = ~~0<1 \text{ always}~~$

$= \lim\left|7x\cdot\frac{n(n-1)\cdots}{(n+1)\cancel{n(n-1)}\cdots}\right| = \lim\left|\frac{7x}{n+1}\right| = 0<1$ always

the series is always converged

$I = (-\infty, \infty)$, $R = \infty$

Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-7x)^n}{n!}$.

Enter INF for ∞ and MINF for $-\infty$.

The series is convergent from $x =$ MINF, left end included (enter Y or N): N

to $x =$ INF, right end included (enter Y or N): N

The radius of convergence is R = INF

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6. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.9.pg

Book Problem 9

$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{\frac{(-4)^{n+1}x^{n+1}}{\sqrt[6]{n+1}}}{\frac{(-4)^n x^n}{\sqrt[6]{n}}}\right| = \left|\frac{4^{n+1}x^{n+1}}{\sqrt[6]{n+1}}\cdot\frac{\sqrt[6]{n}}{4^n x^n}\right| = \left|4x\sqrt[6]{\frac{n}{n+1}}\right| \Rightarrow \lim\left|\frac{a_{n+1}}{a_n}\right| = |4x| < 1$

$|x| < \frac{1}{4} \to R$

$\boxed{-\frac{1}{4} < x \le \frac{1}{4}}$

Find the interval of convergence of the series $\sum_{n=1}^{\infty} \frac{(-4)^n x^n}{\sqrt[6]{n}}$.

if $x = \frac{1}{4}$: $\sum\frac{(-4)^n(1/4)^n}{\sqrt[6]{n}} = \sum\frac{(-1)^n}{\sqrt[6]{n}}$ conv alt series

if $x = -\frac{1}{4}$: $\sum\frac{(-4)^n(-1/4)^n}{\sqrt[6]{n}} = \sum\frac{1}{\sqrt[6]{n}}$ Div p-series

The series is convergent from $x = -1/4$, left end included (enter Y or N): N

to $x = 1/4$, right end included (enter Y or N): Y

The radius of convergence is R = 1/4

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7. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.11.pg

Book Problem 11

Find the interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^n\frac{x^n}{10^n(\ln n)}$.

The series is convergent from $x =$ ___, left end included (enter Y or N): ___

to $x =$ ___, right end included (enter Y or N): ___

The radius of convergence is R = ___

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8. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.13.pg

Book Problem 13

$$\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{(-1)^{n+1}(x+9)^{n+1}}{(n+1)6^{n+1}} \cdot \frac{n6^n}{(-1)^n(x+9)^n}\right| = \left|\frac{(x+9)}{6}\cdot\frac{n}{n+1}\right|, \quad \frac{n}{n+1}\to 1$$

Find the interval of convergence of the series $\sum_{n=1}^{\infty}(-1)^n\frac{(x+9)^n}{n(6)^n}$.

$$\lim\left|\frac{a_{n+1}}{a_n}\right| = \left|\frac{x+9}{6}\right| < 1 \Rightarrow |x+9| < 6 \to R$$

The series is convergent from $x =$ <u>-15</u>, left end included (enter Y or N): <u>N</u>

to $x =$ <u>-3</u>, right end included (enter Y or N): <u>Y</u>

The radius of convergence is R = <u>6</u>.

If $x=-15$: $\sum \frac{(-1)^n(-6)^n}{n(6^n)} = \sum \frac{1}{n}$ div harmonic

If $x=-3$: $\sum \frac{(-1)^n(-3+9)^n}{n(6^n)} = \sum \frac{(-1)^n 6^n}{n(6^n)} = \sum \frac{(-1)^n}{n}$ conv altseries

$-6 < x+9 < 6$

$\boxed{-15 < x \leq -3}$

$-15 \quad -9 \quad -3$

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9. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.14.pg

Book Problem 14

Find the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{(-9)^n}{\sqrt{n}}(x+4)^n$.

The series is convergent from $x =$ _____, left end included (enter Y or N): ___

to $x =$ _____, right end included (enter Y or N): ___

The radius of convergence is R = ___.

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10. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.15.pg

Book Problem 15

$$\lim\left|\frac{a_{n+1}}{a_n}\right| = \lim\left|\frac{n+1}{7^{n+1}}(2x-9)^{n+1}\cdot\frac{7^n}{n(2x-9)^n}\right| = \lim_{n\to\infty}\left|\frac{n+1}{n}\cdot\frac{(2x-9)}{7}\right| = \left|\frac{2x-9}{7}\right| < 1, \quad \frac{n+1}{n}\to 1$$

Find the interval of convergence of the power series $\sum_{n=1}^{\infty}\frac{n}{7^n}(2x-9)^n$.

$$|2x-9| < 7 \Rightarrow -7 < 2x-9 < 7 \Rightarrow 2 < 2x < 16$$

The series is convergent from $x =$ <u>1</u>, left end included (enter Y or N): <u>N</u>

to $x =$ <u>8</u>, right end included (enter Y or N): <u>N</u>

The radius of convergence is R = <u>3.5</u>

If $x=1$: $\sum \frac{n(-7)^n}{7^n} = \sum(-1)^n n$ Divergent by the test for Div

$\Rightarrow \boxed{1 < x < 8}$

If $x=8$: $\sum \frac{n(7)^n}{7^n} = \sum n$ Div

$R = 3.5$

$1 \quad 4.5 \quad 8$

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11. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.17.pg

Book Problem 17

$$\left|\frac{a_{n+1}}{a_n}\right| = \frac{(n+1)!\,x^{n+1}}{n!\,x^n} = |(n+1)x|, \quad \lim\left|\frac{a_{n+1}}{a_n}\right| = \begin{cases}0 & \text{if } x=0\\ \infty & \text{if } x\neq 0\end{cases}$$

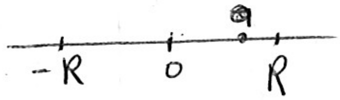
A) $\sum_{n=1}^{\infty} n!\,x^n$ is convergent for $x =$ <u>0</u>. the series is convergent for only $x=0$

B) $\sum_{n=1}^{\infty} n!\,(6x-6)^n$ is convergent for $x =$ <u>6</u>.

C) The radius of convergence of both series is R = <u>0</u> (there is only 1 point of convergence)

12. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.19.pg

Book Problem 19



If $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = 7$, does it follow that the following series are convergent? → Series converges on $(-7,7)$ for sure

A) $\sum_{n=0}^{\infty} c_n(-6)^n$ [?] Yes

B) $\sum_{n=0}^{\infty} c_n(-7)^n$ [?] No

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13. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.20.pg

Book Problem 20

→ series converges on $(-6,6)$ for sure

Suppose that $\sum_{n=0}^{\infty} c_n x^n$ converges when $x = -6$ and diverges when $x = 7$. What can be said about the convergence or divergence of the following series? series diverges outside $[-7,7]$ for sure.

A) $\sum_{n=0}^{\infty} c_n$ [?] ~~Yes~~ convergent since here $x=1$

B) $\sum_{n=0}^{\infty} c_n(9)^n$ [?] ~~No~~ divergent → since $x = 9 > 7$

C) $\sum_{n=0}^{\infty} c_n(-4)^n$ [?] ~~Yes~~ convergent

D) $\sum_{n=0}^{\infty} (-1)^n c_n 11^n$ [?] $\Sigma\, c_n(-11)^n$ divergent since $x = -11 < -7$

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14. (1 pt) UNCC1242/EssentialCalculus-Stewart-Sec8.5.Extra.pg

Extra Problem

Match each of the power series with its interval of convergence.

___1. $\sum_{n=1}^{\infty} \frac{n!(9x-5)^n}{5^n}$

___2. $\sum_{n=1}^{\infty} \frac{(9x)^n}{n^5}$

___3. $\sum_{n=1}^{\infty} \frac{(x-5)^n}{(5)^n}$

___4. $\sum_{n=1}^{\infty} \frac{(x-5)^n}{(n!)5^n}$

A. $[\frac{-1}{9}, \frac{1}{9}]$
B. $(0, 10)$
C. $(-\infty, \infty)$
D. $\{5/9\}$