The Integral & P-Series Tests

sections 8.3

The Integral Test:
$\sum_{n=k}^{\infty} a_n$ converges/diverges exactly when $\int_k^{\infty} f(x)\mathrm{d}x$ converges/diverges, where $f(n) = a_n$.

Examples:

1. $\sum_{n=1}^{\infty} \frac{1}{n^2}$

2. $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$

3. $\sum_{n=1}^{\infty} \frac{1}{n}$

4. $\sum_{n=3}^{\infty} \frac{1}{n\ln(n)}$

5. $\sum_{n=1}^{\infty} ne^{-n^2}$

The Form/Definition of a P-Series:
$\sum \frac{c}{n^p}$, where c and p are constants. Note, if p is negative, we don't have a fraction.

The P-Series Test (a special case of the integral test):
If $p > 1$, the series will converge, and if $p \leq 1$, the series diverges.

Examples:

1. $\sum_{n=1}^{\infty} \frac{4}{n^{2/3}}$

2. $\sum_{n=2}^{\infty} \frac{-2}{\sqrt[5]{n^6}}$

3. $\sum_{n=4}^{\infty} 5n^3$