Properties of Summation
sections 5.1 - 5.2 AND 8.2

The notation of the summation:

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_{n-1} + a_n$$

The symbol a_i is a special type of function, where i is what is plugged into the function (but i is only allowed to be an integer). The sum $\sum_{i=1}^{n} a_i$ tells you to plug in $i = 1$ (below the sigma) and all of the integers up to $i = n$ (above the sigma) into the formula a_i. Then add up all of those numbers. Hence the name: summation.

For **Finite Sums**, we have the following properties:

$$\sum_{k=1}^{n} c \overset{\text{OR}}{=} \sum_{i=1}^{n} c \overset{\text{OR}}{=} c + c + c + \ldots + c = c \cdot n$$

$$\sum_{k=1}^{n} k \overset{\text{OR}}{=} \sum_{i=1}^{n} i \overset{\text{OR}}{=} 1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^{n} k^2 \overset{\text{OR}}{=} \sum_{i=1}^{n} i^2 \overset{\text{OR}}{=} 1^2 + 2^2 + 3^2 + \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^{n} k^3 \overset{\text{OR}}{=} \sum_{i=1}^{n} i^3 \overset{\text{OR}}{=} 1^3 + 2^3 + 3^3 + \ldots + n^3 = \left[\frac{n(n+1)}{2}\right]^2$$

AND

$$\sum_{k=1}^{n} c \cdot a_k = c \cdot \sum_{k=1}^{n} a_k$$

$$\sum_{k=1}^{n} (a_k + b_k) = \sum_{k=1}^{n} a_k + \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} (a_k - b_k) = \sum_{k=1}^{n} a_k - \sum_{k=1}^{n} b_k$$

BUT

$$\sum_{k=1}^{n} a_k \cdot b_k \neq \sum_{k=1}^{n} a_k \cdot \sum_{k=1}^{n} b_k$$

$$\sum_{k=1}^{n} \frac{a_k}{b_k} \neq \frac{\sum_{k=1}^{n} a_k}{\sum_{k=1}^{n} b_k}$$

For **Infinite Sums**, the above relationships are true **if and only if** the series coverges, where

$$\sum_{n=1}^{\infty} a_n = \sum_{k=1}^{\infty} a_k = \lim_{n\to\infty} \sum_{k=1}^{n} a_k$$