The General Arctan Rule

Completing the Square:

This is a technique to rewrite a polynomial into the form of $(polynomial)^2 + (constant)$
Start with a polynomial: $x^2 + bx + c$. Note, you cannot have a leading coefficient, so if you do have one, factor out by that coefficient first. Then add a constant to create a square polynomial:

$$x^2 + bx + c = \left(x^2 + bx + \left(\frac{b}{2}\right)^2\right) + c - \left(\frac{b}{2}\right)^2 = \left(x + \frac{b}{2}\right)^2 + c - \frac{b^2}{4}$$

You'll need to also minus what you added in order to stay equal to the polynomial you started with. Automatically, you'll get the form we want. This is because of the rule: $(x+a)^2 = x^2 + 2ax + a^2$.

Example: complete the square on $x^2 - 4x + 8$

$$\begin{aligned} x^2 - 4x + 8 &= (x^2 - 4x \qquad) + 8 \\ &= \left(x^2 - 4x + \left(-\frac{4}{2}\right)^2\right) + 8 - \left(-\frac{4}{2}\right)^2 \\ &= (x^2 - 4x + 4) + 8 - 4 \\ &= (x-2)^2 + 4 \end{aligned}$$

Example: complete the square on $5x^2 + 10x + 14$

$$\begin{aligned} 5x^2 + 10x + 14 &= 5(x^2 + 2x \qquad) + 14 \\ &= 5\left(x^2 + 2x + \left(\frac{2}{2}\right)^2\right) + 14 - 5\left(\frac{2}{2}\right)^2 \\ &= 5(x^2 + 2x + 1) + 14 - 5 \\ &= 5(x+1)^2 + 9 \end{aligned}$$

The General Arctan Rule:

The Rule: $\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\arctan\left(\frac{x}{a}\right) + C$

OR $\int \frac{g'(x)}{a^2+(g(x))^2}\,dx = \frac{1}{a}\arctan\left(\frac{g(x)}{a}\right) + C$

Putting everything together:

Use this rule when you have a fraction of the form: $\frac{poly}{poly}$ where the polynomial in the denominator does not factor and the fraction is not in the correct form to turn into an $ln(\,)$.

Example: evaluate $\int \frac{2}{x^2-4x+8}\,dx$

From the example above we know that $x^2 - 4x + 8 = (x-2)^2 + 4$. Thus, this integral can be rewritten as

$$\int \frac{2}{x^2 - 4x + 8}\,dx = \int \frac{2}{(x-2)^2 + 4}\,dx = 2\left[\frac{1}{2}\arctan\left(\frac{x-2}{2}\right)\right] + C = \arctan\left(\frac{x-2}{2}\right) + C$$