Application to Dynamical Systems:

example (from sect 1.10):

$$A = \begin{bmatrix} .94 & .04 \\ .06 & .96 \end{bmatrix} \qquad \bar{X}_0 = \begin{bmatrix} 10 \\ 8 \end{bmatrix}$$ units = millions of people

step 1: find eigenvalues

$$0 = \det(A - \lambda I) = \begin{vmatrix} .94-\lambda & .04 \\ .06 & .96-\lambda \end{vmatrix} = (.94-\lambda)(.96-\lambda) - .0024$$

$$= .9024 \qquad \lambda + \lambda^2 - .0024$$

$$0 = .9 - 1.9\lambda + \lambda^2$$

$$\Rightarrow \lambda = 1 \;,\; 0.9$$

step 2: find the corresponding eigenvectors

$$\lambda_1 = 1 \Longrightarrow \qquad \bar{V}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\lambda_2 = .9 \Longrightarrow \qquad \bar{V}_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

Step 3: write $\bar{X}_0$ as a linear combin of $\bar{V}_1, \bar{V}_2$

want $\quad \bar{X}_0 = C_1\bar{V}_1 + C_2\bar{V}_2$

$$\left[\begin{array}{cc|c} 2 & 1 & 10 \\ 3 & -1 & 8 \end{array}\right] \xrightarrow{\text{ref}} \left[\begin{array}{cc|c} 1 & 0 & 3.6 \\ 0 & 1 & 2.8 \end{array}\right]$$

$C_1 = 3.6$

$C_2 = 2.8$

←

step 4: compute $\bar{X}_k$

$$\bar{X}_1 = A\bar{x}_0 = A(c_1\bar{v}_1 + c_2\bar{v}_2)$$

$$= c_1 \cdot A\bar{v}_1 + c_2 \cdot A\bar{v}_2$$

$$= c_1 \cdot \lambda_1\bar{v}_1 + c_2 \cdot \lambda_2\bar{v}_2$$

$$\bar{X}_2 = A\bar{X}_1 = A(c_1\lambda_1\bar{v}_1 + c_2\lambda_2\bar{v}_2)$$

$$= c_1\lambda_1 \cdot A\bar{v}_1 + c_2\lambda_2 \cdot A\bar{v}_2$$

$$= c_1\lambda_1^2\bar{v}_1 + c_2\lambda_2^2\bar{v}_2$$

$$\vdots$$

$$\bar{X}_k = c_1 \cdot \lambda_1^k \cdot \bar{v}_1 + c_2 \cdot \lambda_2^k \bar{v}_2$$

pluging in what we know:

$$\bar{X}_k = (3.6)\begin{bmatrix} 2 \\ 3 \end{bmatrix} + (2.8)(.9)^k\begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

<u>Result</u>: as $k \to \infty$, $(.9)^k \to 0$

as $k \uparrow$, $\bar{X}_k \longrightarrow (3.6)\begin{bmatrix} 2 \\ 3 \end{bmatrix} = \begin{bmatrix} 7.2 \\ 10.8 \end{bmatrix}$