1.10 EXERCISES

1. The container of a breakfast cereal usually lists the number of calories and the amounts of protein, carbohydrate, and fat contained in one serving of the cereal. The amounts for two common cereals are given below. Suppose a mixture of these two cereals is to be prepared that contains exactly 295 calories, 9 g of protein, 48 g of carbohydrate, and 8 g of fat.

   a. Set up a vector equation for this problem. Include a statement of what the variables in your equation represent.

   b. Write an equivalent matrix equation, and then determine if the desired mixture of the two cereals can be prepared.

Nutrition Information per Serving

Nutrient	General Mills Cheerios®	Quaker® 100% Natural Cereal
Calories	110	130
Protein (g)	4	3
Carbohydrate (g)	20	18
Fat (g)	2	5

2. One serving of Shredded Wheat supplies 160 calories, 5 g of protein, 6 g of fiber, and 1 g of fat. One serving of Crispix® supplies 110 calories, 2 g of protein, .1 g of fiber, and .4 g of fat.

   a. Set up a matrix B and a vector $\mathbf{u}$ such that $B\mathbf{u}$ gives the amounts of calories, protein, fiber, and fat contained in a mixture of three servings of Shredded Wheat and two servings of Crispix.

   b. [M] Suppose that you want a cereal with more fiber than Crispix but fewer calories than Shredded Wheat. Is it possible for a mixture of the two cereals to supply 130 calories, 3.20 g of protein, 2.46 g of fiber, and .64 g of fat? If so, what is the mixture?

3. After taking a nutrition class, a big Annie's® Mac and Cheese fan decides to improve the levels of protein and fiber in her favorite lunch by adding broccoli and canned chicken. The nutritional information for the foods referred to in this exercise are given in the table below.

Nutrition Information per Serving

| Nutrient	Mac and Cheese	Broccoli	Chicken	Shells
Calories	270	51	70	260
Protein (g)	10	5.4	15	9
Fiber (g)	2	5.2	0	5

   a. [M] If she wants to limit her lunch to 400 calories but get 30 g of protein and 10 g of fiber, what proportions of servings of Mac and Cheese, broccoli, and chicken should she use?

   b. [M] She found that there was too much broccoli in the proportions from part (a), so she decided to switch from classical Mac and Cheese to Annie's® Whole Wheat Shells and White Cheddar. What proportions of servings of each food should she use to meet the same goals as in part (a)?

4. The Cambridge Diet supplies .8 g of calcium per day, in addition to the nutrients listed in the Table 1 for Example 1. The amounts of calcium per unit (100 g) supplied by the three ingredients in the Cambridge Diet are as follows: 1.26 g from nonfat milk, .19 g from soy flour, and .8 g from whey. Another ingredient in the diet mixture is isolated soy protein, which provides the following nutrients in each unit: 80 g of protein, 0 g of carbohydrate, 3.4 g of fat, and .18 g of calcium.

   a. Set up a matrix equation whose solution determines the amounts of nonfat milk, soy flour, whey, and isolated soy protein necessary to supply the precise amounts of protein, carbohydrate, fat, and calcium in the Cambridge Diet. State what the variables in the equation represent.

   b. [M] Solve the equation in (a) and discuss your answer.

In Exercises 5–8, write a matrix equation that determines the loop currents. [M] If MATLAB or another matrix program is available, solve the system for the loop currents.

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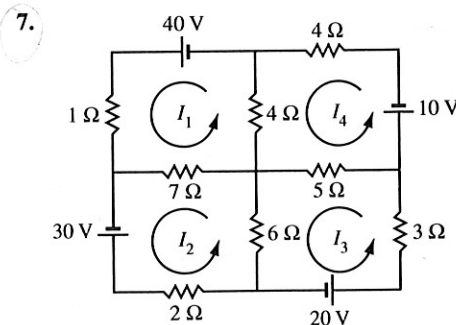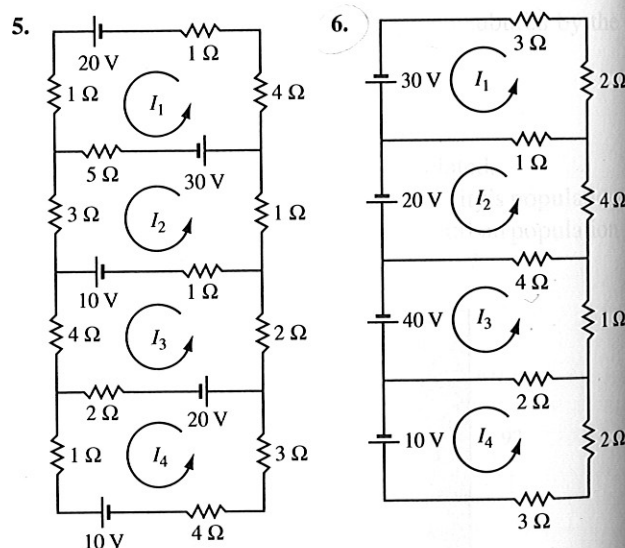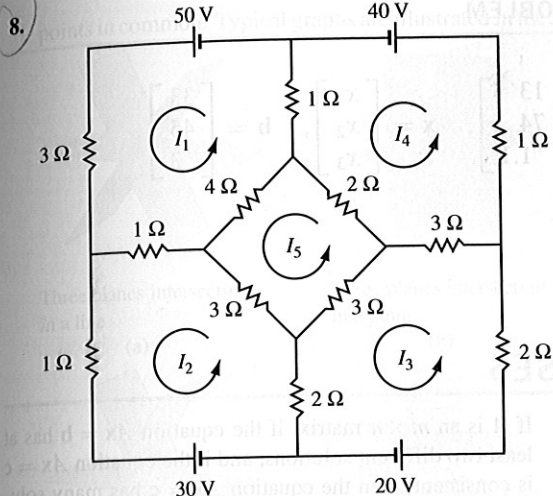
9. In a certain region, about 7% of a city's population moves to the surrounding suburbs each year, and about 5% of the suburban population moves into the city. In 2010, there were 800,000 residents in the city and 500,000 in the suburbs. Set up a difference equation that describes this situation, where x_0 is the initial population in 2010. Then estimate the populations in the city and in the suburbs two years later, in 2012. (Ignore other factors that might influence the population sizes.)

10. In a certain region, about 6% of a city's population moves to the surrounding suburbs each year, and about 4% of the suburban population moves into the city. In 2010, there were 10,000,000 residents in the city and 800,000 in the suburbs. Set up a difference equation that describes this situation, where x_0 is the initial population in 2010. Then estimate the populations in the city and in the suburbs two years later, in 2012.

11. In 1994, the population of California was 31,524,000, and the population living in the United States but *outside* California was 228,680,000. During the year, it is estimated that 516,100 persons moved from California to elsewhere in the United States, while 381,262 persons moved to California from elsewhere in the United States.[4]

a. Set up the migration matrix for this situation, using five decimal places for the migration rates into and out of California. Let your work show how you produced the migration matrix.

b. [M] Compute the projected populations in the year 2000 for California and elsewhere in the United States, assuming that the migration rates did not change during the 6-year period. (These calculations do not take into account births, deaths, or the substantial migration of persons into California and elsewhere in the United States from other countries.)

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[4] Migration data supplied by the Demographic Research Unit of the California State Department of Finance.

12. [M] Budget® Rent A Car in Wichita, Kansas has a fleet of about 500 cars, at three locations. A car rented at one location may be returned to any of the three locations. The various fractions of cars returned to the three locations are shown in the matrix below. Suppose that on Monday there are 295 cars at the airport (or rented from there), 55 cars at the east side office, and 150 cars at the west side office. What will be the approximate distribution of cars on Wednesday?

Cars Rented From:

Airport	East	West	Returned To:
.97	.05	.10	Airport
.00	.90	.05	East
.03	.05	.85	West

13. [M] Let M and x_0 be as in Example 3.
a. Compute the population vectors x_k for $k = 1, \ldots, 20$. Discuss what you find.
b. Repeat part (a) with an initial population of 350,000 in the city and 650,000 in the suburbs. What do you find?

14. [M] Study how changes in boundary temperatures on a steel plate affect the temperatures at interior points on the plate.
a. Begin by estimating the temperatures T_1, T_2, T_3, T_4 at each of the sets of four points on the steel plate shown in the figure. In each case, the value of T_k is approximated by the average of the temperatures at the four closest points. See Exercises 33 and 34 in Section 1.1, where the values (in degrees) turn out to be $(20, 27.5, 30, 22.5)$. How is this list of values related to your results for the points in set (a) and set (b)?
b. Without making any computations, guess the interior temperatures in (a) when the boundary temperatures are all multiplied by 3. Check your guess.
c. Finally, make a general conjecture about the correspondence from the list of eight boundary temperatures to the list of four interior temperatures.

(a)

(b)