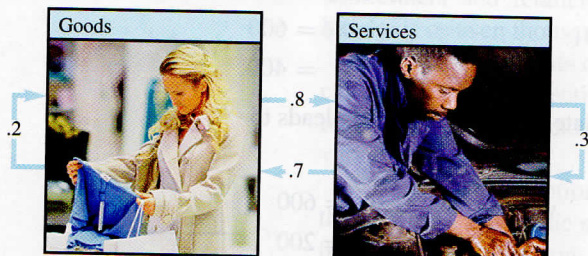


## 1.6 EXERCISES

1. Suppose an economy has only two sectors: Goods and Services. Each year, Goods sells 80% of its output to Services and keeps the rest, while Services sells 70% of its output to Goods and retains the rest. Find equilibrium prices for the annual outputs of the Goods and Services sectors that make each sector's income match its expenditures.



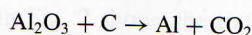
2. Find another set of equilibrium prices for the economy in Example 1. Suppose the same economy used Japanese yen instead of dollars to measure the values of the various sectors' outputs. Would this change the problem in any way? Discuss.
3. Consider an economy with three sectors: Fuels and Power, Manufacturing, and Services. Fuels and Power sells 80% of its output to Manufacturing, 10% to Services, and retains the rest. Manufacturing sells 10% of its output to Fuels and Power, 80% to Services, and retains the rest. Services sells 20% to Fuels and Power, 40% to Manufacturing, and retains the rest.
- Construct the exchange table for this economy.
  - Develop a system of equations that leads to prices at which each sector's income matches its expenses. Then write the augmented matrix that can be row reduced to find these prices.
  - [M] Find a set of equilibrium prices when the price for the Services output is 100 units.
4. Suppose an economy has four sectors: Mining, Lumber, Energy, and Transportation. Mining sells 10% of its output to Lumber, 60% to Energy, and retains the rest. Lumber sells 15% of its output to Mining, 50% to Energy, 20% to Transportation, and retains the rest. Energy sells 20% of its output to Mining, 15% to Lumber, 20% to Transportation, and retains the rest. Transportation sells 20% of its output to Mining, 10% to Lumber, 50% to Energy, and retains the rest.
- Construct the exchange table for this economy.
  - [M] Find a set of equilibrium prices for the economy.
5. An economy has four sectors: Agriculture, Manufacturing, Services, and Transportation. Agriculture sells 20% of its output to Manufacturing, 30% to Services, 30% to Transportation, and retains the rest. Manufacturing sells 35% of its output to Agriculture, 35% to Services, 20% to Transportation, and retains the rest. Services sells 10% of its output to Agriculture, 20% to Manufacturing, 20% to Transportation,

and retains the rest. Transportation sells 20% of its output to Agriculture, 30% to Manufacturing, 20% to Services, and retains the rest.

- Construct the exchange table for this economy.
- [M] Find a set of equilibrium prices for the economy if the value of Transportation is \$10.00 per unit.
- The Services sector launches a successful "eat farm fresh" campaign, and increases its share of the output from the Agricultural sector to 40%, whereas the share of Agricultural production going to Manufacturing falls to 10%. Construct the exchange table for this new economy.
- [M] Find a set of equilibrium prices for this new economy if the value of Transportation is still \$10.00 per unit. What effect has the "eat farm fresh" campaign had on the equilibrium prices for the sectors in this economy?

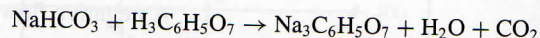
Balance the chemical equations in Exercises 6–11 using the vector equation approach discussed in this section.

6. Aluminum oxide and carbon react to create elemental aluminum and carbon dioxide:

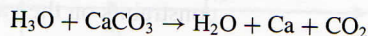


[For each compound, construct a vector that lists the number of atoms of aluminum, oxygen, and carbon.]

7. Alka-Seltzer contains sodium bicarbonate ( $\text{NaHCO}_3$ ) and citric acid ( $\text{H}_3\text{C}_6\text{H}_5\text{O}_7$ ). When a tablet is dissolved in water, the following reaction produces sodium citrate, water, and carbon dioxide (gas):



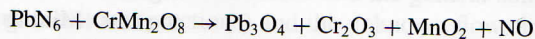
8. Limestone,  $\text{CaCO}_3$ , neutralizes the acid,  $\text{H}_3\text{O}^+$ , in acid rain by the following unbalanced equation:



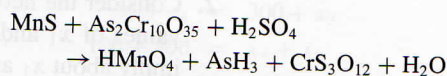
9. Boron sulfide reacts violently with water to form boric acid and hydrogen sulfide gas (the smell of rotten eggs). The unbalanced equation is



10. [M] If possible, use exact arithmetic or a rational format for calculations in balancing the following chemical reaction:

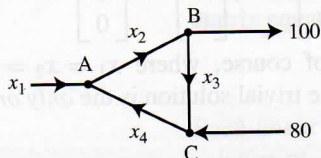


11. [M] The chemical reaction below can be used in some industrial processes, such as the production of arsene ( $\text{AsH}_3$ ). Use exact arithmetic or a rational format for calculations to balance this equation.

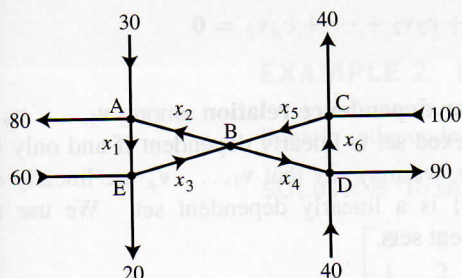




- Find the general flow pattern of the network shown in the figure. Assuming that the flows are all nonnegative, what is the smallest possible value for  $x_4$ ?



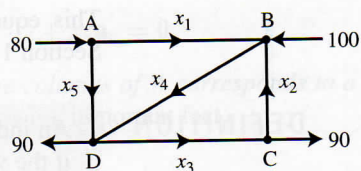
- a. Find the general flow pattern of the network shown in the figure.
- b. Assuming that the flow must be in the directions indicated, find the minimum flows in the branches denoted by  $x_2$ ,  $x_3$ ,  $x_4$ , and  $x_5$ .



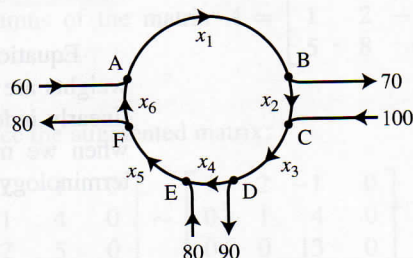
- a. Find the general traffic pattern of the freeway network

shown in the figure. (Flow rates are in cars/minute.)

- b. Describe the general traffic pattern when the road whose flow is  $x_5$  is closed.
- c. When  $x_5 = 0$ , what is the minimum value of  $x_4$ ?



15. Intersections in England are often constructed as one-way "roundabouts," such as the one shown in the figure. Assume that traffic must travel in the directions shown. Find the general solution of the network flow. Find the smallest possible value for  $x_6$ .



## SOLUTIONS TO PRACTICE PROBLEMS

1. Write the percentages as decimals. Since all output must be taken into account, each column must sum to 1. This fact helps to fill in any missing entries.

### Distribution of Output from:

Agriculture	Mining	Manufacturing	Purchased by:
.65	.20	.20	Agriculture
.05	.10	.30	Mining
.30	.70	.50	Manufacturing

2. Since  $x_5 \leq 500$ , the equations D and A for  $x_1$  and  $x_2$  imply that  $x_1 \geq 100$  and  $x_2 \leq 700$ . The fact that  $x_5 \geq 0$  implies that  $x_1 \leq 600$  and  $x_2 \geq 200$ . So,  $100 \leq x_1 \leq 600$ , and  $200 \leq x_2 \leq 700$ .

## 1.7 LINEAR INDEPENDENCE

The homogeneous equations in Section 1.5 can be studied from a different perspective by writing them as vector equations. In this way, the focus shifts from the unknown solutions of  $Ax = 0$  to the vectors that appear in the vector equations.