

4. Find the pt of intersection:

$$x_1 + 2x_2 = -13; 3x_1 - 2x_2 = 1$$

$$x_1 = -2x_2 - 13$$

$$\Rightarrow 3(-2x_2 - 13) - 2x_2 = 1$$

$$-8x_2 - 39 = 1$$

$$x_2 = -\frac{40}{8} = -5$$

$$x_1 = 10 - 13 = -3$$

∴ point is (-3, -5)

For 11-12, Solve the systems.

11. $\begin{cases} x_2 + 5x_3 = -4 \\ x_1 + 4x_2 + 3x_3 = -2 \\ 2x_1 + 7x_2 + x_3 = -2 \end{cases}$

$$\sim \left[\begin{array}{ccc|c} 0 & 1 & 5 & -4 \\ 1 & 4 & 3 & -2 \\ 2 & 7 & 1 & -2 \end{array} \right] \begin{matrix} R_1 \leftrightarrow R_2 \\ R_3 \leftrightarrow R_3 - 2R_2 \end{matrix} \sim \left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & -1 & -5 & 2 \end{array} \right] \begin{matrix} R_3 + R_2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & 4 & 3 & -2 \\ 0 & 1 & 5 & -4 \\ 0 & 0 & 0 & -2 \end{array} \right] \rightarrow 0 = -2$$

So, there's no solution

12. $\begin{cases} x_1 - 5x_2 + 4x_3 = -3 \\ 2x_1 - 7x_2 + 3x_3 = -2 \\ -2x_1 + x_2 + 7x_3 = -1 \end{cases}$

$$\sim \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 2 & -7 & 3 & -2 \\ -2 & 1 & 7 & -1 \end{array} \right] \begin{matrix} R_2 \leftrightarrow R_2 - 2R_1 \\ R_3 \leftrightarrow R_3 + R_2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & -6 & 10 & -3 \end{array} \right] \begin{matrix} R_3 \leftrightarrow R_3 + 2R_2 \end{matrix}$$

$$\sim \left[\begin{array}{ccc|c} 1 & -5 & 4 & -3 \\ 0 & 3 & -5 & 4 \\ 0 & 0 & 0 & 5 \end{array} \right] \rightarrow 0 = 5$$

So no solution

17. $\begin{cases} 2x_1 + 3x_2 = -1 \\ 6x_1 + 5x_2 = 0 \\ 2x_1 - 5x_2 = 7 \end{cases}$ have a common point of intersection?

$$2x_1 = 5x_2 + 7$$

$$\Rightarrow (5x_2 + 7) + 3x_2 = -1 \text{ AND } 3(5x_2 + 7) + 5x_2 = 0$$

$$8x_2 = -8$$

$$x_2 = -1$$

$$20x_2 + 21 = 0$$

$$x_2 = -\frac{21}{20}$$

7. Continue row operations on the augmented matrix.
Describe the solution set.

$$\left[\begin{array}{cccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & -2 \end{array} \right] \begin{matrix} R_3 \leftrightarrow R_4 \end{matrix} \left[\begin{array}{cccc|c} 1 & 7 & 3 & -4 \\ 0 & 1 & 1 & 3 \\ 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 1 \end{array} \right] \rightarrow 0 = 1$$

Since we have that $0 = 1$, there is no solution to this system.

10. $\left[\begin{array}{ccccc} 1 & 3 & 0 & -2 & -7 \\ 0 & 1 & 0 & 3 & 6 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right] \begin{matrix} R_1 \leftrightarrow R_1 + 2R_4 \\ R_2 \leftrightarrow R_2 - 3R_4 \end{matrix} \sim \left[\begin{array}{ccccc} 1 & 3 & 0 & 0 & -11 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right] R_1 - 3R_2$

$$\sim \left[\begin{array}{ccccc} 1 & 0 & 0 & 0 & -47 \\ 0 & 1 & 0 & 0 & 12 \\ 0 & 0 & 1 & 0 & 2 \\ 0 & 0 & 0 & 1 & -2 \end{array} \right] \Rightarrow \begin{matrix} x_1 = -47 \\ x_2 = 12 \\ x_3 = 2 \\ x_4 = -2 \end{matrix}$$

16. determine if the system is consistent.

$$\begin{cases} 2x_1 - 6x_2 = 5 \\ x_2 - 4x_3 + x_4 = 0 \\ -x_1 + 6x_2 + x_3 + 5x_4 = 3 \\ -x_2 + 5x_3 + 4x_4 = 0 \end{cases}$$

$$\sim \left[\begin{array}{cccc|c} 2 & -6 & 0 & 0 & 5 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & -1 & 5 & 4 & 0 \end{array} \right] \begin{matrix} R_1 + R_3 \\ R_4 + R_2 \end{matrix} \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 5 & 8 \\ 0 & 1 & -4 & 1 & 0 \\ -1 & 6 & 1 & 5 & 3 \\ 0 & 0 & 1 & 5 & 0 \end{array} \right] \begin{matrix} R_3 \leftrightarrow R_3 + R_1 \\ R_4 \leftrightarrow R_3 \end{matrix}$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 5 & 8 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 1 & 5 & 0 \end{array} \right] \sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 5 & 8 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 11 \end{array} \right] R_4 - 26R_3$$

$$\sim \left[\begin{array}{cccc|c} 1 & 0 & 1 & 5 & 8 \\ 0 & 1 & -4 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 0 & 11 \end{array} \right]$$

consistent

For 21-22, find k so that the system is consistent.

21. $\left[\begin{array}{ccc|c} 1 & 4 & -2 \\ 3 & k & -6 \end{array} \right] R_2 \leftrightarrow R_2 - 3R_1 \sim \left[\begin{array}{ccc|c} 1 & 4 & -2 \\ 0 & k-12 & 0 \end{array} \right]$

any k
would give
a consistent
system

22. $\left[\begin{array}{ccc|c} -4 & 12 & h \\ 2 & -6 & -3 \end{array} \right] R_1 \leftrightarrow R_1 + 2R_2 \sim \left[\begin{array}{ccc|c} 0 & 0 & h-6 \\ 2 & -6 & -3 \end{array} \right]$

here $h=6$ will give a consistent system.

So, no common point of intersection

23. a) every row operation is reversible.

true, p6

b) a 5×6 matrix has 6 rows
false, it has 5 rows, 6 columns

c) the solution set of a system with variables x_1, \dots, x_n is a list of #s (s_1, \dots, s_n) such that $x_1 = s_1, \dots, x_n = s_n$.
false, the solution set is a set of these solutions not just 1 like described here

d) 2 Questions: Uniqueness & Existence
true, p7

25. find g, h, k to be consistent

$$\begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ -2 & 5 & -9 & k \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & -3 & 5 & k-2g \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -4 & 7 & g \\ 0 & 3 & -5 & h \\ 0 & 0 & 0 & k+h-2g \end{bmatrix} \Rightarrow \text{need: } k+h-2g=0$$

$R_3 \leftrightarrow R_3 + R_2$

For 29-30, find the row operation that transforms the matrices into each other

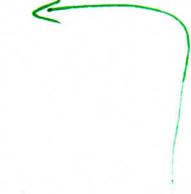
29. $\begin{bmatrix} 0 & -2 & 5 \\ 1 & 3 & -5 \\ 3 & -1 & 6 \end{bmatrix} \quad R_1 \leftrightarrow R_3 \quad \begin{bmatrix} 3 & -1 & 6 \\ 1 & 3 & -5 \\ 0 & -2 & 5 \end{bmatrix}$

30. $\begin{bmatrix} 1 & 3 & -4 \\ 0 & -2 & 6 \\ 0 & -5 & 10 \end{bmatrix} \quad R_3 \leftrightarrow -\frac{1}{5}R_3 \quad \sim \quad \begin{bmatrix} 1 & 3 & -4 \\ 0 & -2 & 6 \\ 0 & 1 & -2 \end{bmatrix}$

and

$$\begin{bmatrix} 1 & 3 & -4 \\ 0 & -2 & 6 \\ 0 & 1 & -2 \end{bmatrix} \quad R_3 \leftrightarrow -5R_3 \quad \sim \quad \begin{bmatrix} 1 & 3 & -4 \\ 0 & -2 & 6 \\ 0 & 5 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 & 50 \\ 0 & 1 & -4 & 0 & -92.5 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix} \quad R_1 \leftrightarrow R_1 - R_3$$



$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 20 \\ 0 & 1 & 0 & 0 & 27.5 \\ 0 & 0 & 1 & 0 & 30 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix}$$

So, $T_1 = 20^\circ$
 $T_2 = 27.5^\circ$
 $T_3 = 30^\circ$
 $T_4 = 22.5^\circ$

24. a) $A \sim B$ if they have the same # of rows.
false, p6

b) elementary row operations don't change the solution set.
true, p6-7

c) 2 equivalent systems can have different solution sets
false, p3

d) a consistent system has 1+ solutions
true, p4

26. The system below is consistent & f.a.g.
what do we know about c & d?

$$\begin{cases} 2x_1 + 4x_2 = f \\ cx_1 + dx_2 = g \end{cases}$$

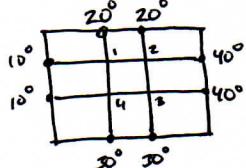
$$\begin{bmatrix} 2 & 4 & f \\ c & d & g \end{bmatrix} \xrightarrow{R_1 \leftrightarrow \frac{1}{2}R_1} \begin{bmatrix} 1 & 2 & \frac{1}{2}f \\ c & d & g \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_2 - CR_1}$$

$$\sim \begin{bmatrix} 1 & 2 & \frac{1}{2}f \\ 0 & d-2c & g-\frac{1}{2}cf \end{bmatrix}$$

case 1: $d-2c \neq 0$ (or $d \neq 2c$)

case 2: $c = \frac{2g}{f}$ & $d = \frac{4g}{f}$

For 33-34, use the following diagram:



33. find temperatures T_1, T_2, T_3, T_4

$$T_1 = (10+20+T_2+T_4)/4 \Rightarrow 4T_1 - T_2 - T_4 = 30$$

$$T_2 = (20+40+T_3+T_1)/4 \Rightarrow -T_1 + 4T_2 - T_3 = 60$$

$$T_3 = (40+30+T_2+T_4)/4 \Rightarrow -T_2 + 4T_3 - T_4 = 70$$

$$T_4 = (10+30+T_1+T_3)/4 \Rightarrow -T_1 - T_3 + 4T_4 = 40$$

34. Solve for T_1, T_2, T_3, T_4 .

$$\begin{bmatrix} 4 & -1 & 0 & -1 & 30 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ -1 & 0 & -1 & 4 & 40 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_4} \sim \begin{bmatrix} -1 & 0 & -1 & 4 & 40 \\ -1 & 4 & -1 & 0 & 60 \\ 0 & -1 & 4 & -1 & 70 \\ 4 & -1 & 0 & -1 & 30 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_2 - R_1}$$

$$\begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 4 & 0 & -4 & 20 \\ 0 & -1 & 4 & -1 & 70 \\ 0 & -1 & -4 & 15 & 190 \end{bmatrix} \xrightarrow{R_2 \leftrightarrow R_2 + 4R_3} \sim \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 0 & 16 & -8 & 300 \\ 0 & 1 & -4 & 1 & -70 \\ 0 & 0 & -8 & 16 & 120 \end{bmatrix} \xrightarrow{R_2 + 2R_4}$$

$$\begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & -4 & 1 & -70 \\ 0 & 0 & -8 & 16 & 120 \\ 0 & 0 & 0 & 24 & 540 \end{bmatrix} \xrightarrow{R_3 \leftrightarrow -\frac{1}{8}R_3} \sim \begin{bmatrix} 1 & 0 & 1 & -4 & -40 \\ 0 & 1 & -4 & 1 & -70 \\ 0 & 0 & 1 & -2 & -15 \\ 0 & 0 & 0 & 1 & 22.5 \end{bmatrix} \xrightarrow{R_4 \leftrightarrow \frac{1}{24}R_4}$$