1. Let $A = \begin{bmatrix} 0 & 2 \\ -7 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 2 & -3 \\ 0 & 1 & -1 \end{bmatrix}$

a) $AB =$

b) $A + B =$

c) $3A - 2B =$

d) $A - C =$

e) $AC =$

f) $C^T =$

2. Find the inverse of the following matrices if they exist.

a) $\begin{bmatrix} 4 & -3 \\ 1 & 7 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & -3 & 0 \\ 0 & 4 & 7 \end{bmatrix}$

3. Solve $A\bar{x} = \bar{b}$ for $\bar{x}$ if $b = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -4 & 3 \\ 2 & 0 & 2 \\ 3 & 7 & -1 \end{bmatrix}$

hint: use A^{-1} if it exists.

4. Suppose you have the following block matrices:

$$A = \left[\begin{array}{cccc|cc} 5 & 7 & 1 & 0 & 3 & 3 \\ -2 & 3 & 4 & 6 & 3 & 3 \\ 4 & 1 & 3 & 4 & 1 & 1 \\ \hline 2 & 3 & -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 1 \end{array}\right] \qquad B = \left[\begin{array}{cc|cccc} 1 & 1 & 0 & 4 & 3 & 5 \\ 4 & 7 & 3 & 1 & 3 & 0 \\ -1 & 0 & 2 & 0 & 4 & -3 \\ \hline 2 & 1 & -1 & -2 & 4 & -1 \\ 3 & 4 & 3 & 4 & 1 & 0 \end{array}\right]$$

a) can you find $A+B$ as block matrices?

b) can you find $A \cdot B$ as block matrices?

5. If you have a block matrix A (see below) how would/should you partition B so that you can multiply AB as block matrices?

$$A = \left[\begin{array}{ccc|cc} 1 & 2 & -1 & 7 & 6 \\ 3 & 1 & 4 & -4 & 0 \\ 5 & 0 & 1 & 2 & -3 \\ \hline 2 & 3 & 3 & 1 & 0 \end{array}\right] \qquad B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -5 & 7 \\ 9 & -8 & 1 \\ 0 & 2 & 7 \\ 1 & 1 & 2 \end{bmatrix}$$

6. Does A have an LU factorization? If so, find it.

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 4 & 1 \\ 10 & 2 & 4 \end{bmatrix}$$

7. Consider $A\bar{x} = \bar{b}$, where $A = \begin{bmatrix} 1 & -3 & 0 & -1 \\ 5 & -17 & 3 & -10 \\ -3 & -7 & 26 & -35 \\ 7 & -27 & 5 & -27 \end{bmatrix}$ & $b = \begin{bmatrix} -1 \\ 1 \\ 51 \\ 12 \end{bmatrix}$

Solve for $\bar{x}$ using the LU factorization of A.

8. find the determinants of the following matrices, using cofactor expansion <u>or</u> the definition of the determinent.

a) $\begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & -3 & 4 \\ 7 & 2 & -2 \\ 4 & -5 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 1 & 0 & 2 & -5 \\ -2 & 3 & 0 & 4 & 7 \\ 4 & 0 & 3 & 2 & -1 \\ 7 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & -4 \end{bmatrix}$

9. find the determinants of the following matrices, using properties of determinants. (hint look at #8)

a) $\begin{bmatrix} 1 & 7 & 4 \\ -3 & 2 & -5 \\ 4 & -2 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 5 & -15 & 20 \\ 7 & 2 & -2 \\ 8 & -10 & 2 \end{bmatrix}$

10. find the determinant, using row ~~operations~~ operations:

$$\begin{vmatrix} 1 & 0 & 3 & -4 \\ 2 & 1 & 4 & 1 \\ -2 & -5 & 2 & 0 \\ 3 & 2 & 1 & -1 \end{vmatrix}$$

11. Solve the following system using row operations, and your answer in parametric vector form:

$$\begin{cases} x + 11y + 2z + w = 9 \\ 2x - 3y - z - 6w = +16 \\ 4x + y - 3z + 5w = 25 \end{cases}$$

12. Determine which of the following sets are linearly independent or linearly dependent.

a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$

e) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\}$

13. Which of the following are linear transformations?

a) $T(x_1, x_2, x_3) = (2x_1 - 3x_2 + x_3,\ x_1 - 4x_3,\ 2x_2)$

b) $T(x_1, x_2) = (x_1 + 3x_2,\ x_1 x_2,\ 3x_2)$

c) $T(x_1, x_2, x_3, x_4) = (3x_2 - x_4,\ x_3 + x_4 + 1,\ -5x_1)$

14. For each mapping in #13, if it's a linear transformation, find it's standard matrix and inverse transformation.

15. The following matrices are the std matrices for a Linear transformation. Determine the domain & codomain of that transformation as well as whether the transformation is 1-to-1 or onto.

a) $\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ 1 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 4 & 0 & 2 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 4 & 0 & 2 \\ 5 & 3 & 2 \\ -1 & 7 & -8 \end{bmatrix}$

d) $\begin{bmatrix} 2 & -3 & 0 \\ -1 & 4 & 5 \\ -3 & 1 & -7 \end{bmatrix}$