

1. Let $A = \begin{bmatrix} 0 & 2 \\ -7 & 4 \end{bmatrix}$, $B = \begin{bmatrix} 1 & -2 \\ 5 & 1 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 2 & -3 \\ 0 & 1 & -1 \end{bmatrix}$

a) $AB = \begin{bmatrix} 10 & 2 \\ 13 & 18 \end{bmatrix}$

b) $A+B = \begin{bmatrix} 1 & 0 \\ -2 & 5 \end{bmatrix}$

c) $3A - 2B = \begin{bmatrix} -2 & 10 \\ -31 & 10 \end{bmatrix}$

d) $A - C = \text{DNE}$

e) $AC = \begin{bmatrix} 0 & 2 & -2 \\ -14 & -10 & 17 \end{bmatrix}$

f) $C^T = \begin{bmatrix} 2 & 0 \\ 2 & 1 \\ -3 & -1 \end{bmatrix}$

2. Find the inverse of the following matrices if they exist.

a) $\begin{bmatrix} 4 & -3 \\ 1 & 7 \end{bmatrix}^{-1} = \frac{1}{28+3} \begin{bmatrix} 7 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 7/31 & 3/31 \\ -1/31 & 4/31 \end{bmatrix}$

b) $\begin{bmatrix} 3 & 2 & 5 \\ 1 & -3 & 0 \\ 0 & 4 & 7 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 \leftrightarrow R_2} \begin{bmatrix} 1 & -3 & 0 & 0 & 1 & 0 \\ 3 & 2 & 5 & 0 & 0 & 0 \\ 0 & 4 & 7 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 = R_2 - 3R_1}$

$\rightarrow \begin{bmatrix} 1 & -3 & 0 & 0 & 1 & 0 \\ 0 & 11 & 5 & 1 & -3 & 0 \\ 0 & 4 & 7 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_2 = \frac{1}{11}} \begin{bmatrix} 1 & -3 & 0 & 0 & 1 & 0 \\ 0 & 1 & 5/11 & 1/11 & -3/11 & 0 \\ 0 & 4 & 7 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_1 + 3R_2} \begin{bmatrix} 1 & 0 & 15/11 & 3/11 & 2/11 & 0 \\ 0 & 1 & 5/11 & 1/11 & -3/11 & 0 \\ 0 & 4 & 7 & 0 & 0 & 1 \end{bmatrix} \xrightarrow{R_3 - 4R_2} \begin{bmatrix} 1 & 0 & 15/11 & 3/11 & 2/11 & 0 \\ 0 & 1 & 5/11 & 1/11 & -3/11 & 0 \\ 0 & 0 & 57/11 & -4/11 & 12/11 & 1 \end{bmatrix} \xrightarrow{\cdot \frac{11}{57}}$

$\rightarrow \begin{bmatrix} 1 & 0 & 15/57 & 3/57 & 2/57 & 0 \\ 0 & 1 & 5/57 & 1/57 & -3/57 & 0 \\ 0 & 0 & 1 & -4/57 & 12/57 & 11/57 \end{bmatrix} \xrightarrow{R_1 - \frac{15}{57}R_3} \begin{bmatrix} 1 & 0 & 0 & 7/19 & -2/19 & -15/57 \\ 0 & 1 & 0 & 7/57 & -3/19 & -5/57 \\ 0 & 0 & 1 & -4/57 & 4/19 & 11/57 \end{bmatrix} \xrightarrow{R_2 - \frac{7}{57}R_3} \Rightarrow \text{inv} = \begin{bmatrix} 7/19 & -3/19 & -15/57 \\ 7/57 & -3/19 & -5/57 \\ -4/57 & 4/19 & 11/57 \end{bmatrix}$

3. Solve $A\bar{x} = \bar{b}$ for $\bar{x}$ if $\bar{b} = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 1 & -4 & 3 \\ 2 & 0 & 2 \\ 3 & 7 & -1 \end{bmatrix}$
 hint: use A^{-1} if it exists.

$\begin{bmatrix} 1 & -4 & 3 & 1 & 0 & 0 \\ 2 & 0 & 2 & 0 & 1 & 0 \\ 3 & 7 & -1 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 & 1 & 0 & 0 \\ 0 & 8 & -4 & -2 & 1 & 0 \\ 0 & 19 & -10 & -3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -4 & 3 & 1 & 0 & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & \frac{1}{8} & 0 \\ 0 & 19 & -10 & -3 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 1 & 0 & \frac{1}{2} & 0 \\ 0 & 1 & -\frac{1}{2} & -\frac{1}{4} & \frac{1}{8} & 0 \\ 0 & 0 & -\frac{1}{2} & \frac{7}{4} & -\frac{19}{8} & 1 \end{bmatrix}$

$\rightarrow \begin{bmatrix} 1 & 0 & 0 & +\frac{7}{2} & +2 \\ 0 & 1 & 0 & -\frac{5}{2} & -1 \\ 0 & 0 & 1 & -\frac{7}{2} & -2 \end{bmatrix}$

So $\bar{x} = A^{-1}\bar{b} = \begin{bmatrix} +\frac{7}{2} & -\frac{17}{4} & +2 \\ -\frac{5}{2} & +\frac{5}{4} & -1 \\ -\frac{7}{2} & +\frac{19}{4} & -2 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 7.5 \\ -4 \\ -7.5 \end{bmatrix}$

4. Suppose you have the following block matrices:

$$A = \begin{bmatrix} 5 & 7 & 1 & 0 & 3 & 3 \\ -2 & 3 & 4 & 6 & 3 & 3 \\ 4 & 1 & 3 & 4 & 1 & 1 \\ \hline 2 & 3 & -1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 1 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 1 & 0 & 4 & 3 & 5 \\ 4 & 7 & 3 & 1 & 3 & 0 \\ -1 & 0 & 2 & 0 & 4 & -3 \\ \hline 2 & 1 & -1 & -2 & 4 & -1 \\ 3 & 4 & 3 & 4 & 1 & 0 \end{bmatrix}$$

a) can you find $A+B$ as block matrices? **NO**

b) can you find $A \cdot B$ as block matrices? **NO**

5. If you have a block matrix A (see below) how would/should you partition B so that you can multiply AB as block matrices?

$$A = \begin{bmatrix} 1 & 2 & -1 & 7 & 6 \\ 3 & 1 & 4 & -4 & 0 \\ 5 & 0 & 1 & 2 & -3 \\ \hline 2 & 3 & 3 & 1 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 3 & 0 \\ 2 & -5 & 7 \\ 9 & -8 & 1 \\ \hline 0 & 2 & 7 \\ 1 & 1 & 2 \end{bmatrix} \leftarrow$$

6. Does A have an LU factorization? If so, find it.

$$A = \begin{bmatrix} 3 & 1 & 4 \\ 2 & 4 & 1 \\ 10 & 2 & 4 \end{bmatrix}$$

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 10/3 & -2/5 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} \textcircled{3} & 1 & 4 \\ 2 & 4 & 1 \\ 10 & 2 & 4 \end{bmatrix}$$

check:

$$L \cdot U = \begin{bmatrix} 1 & 0 & 0 \\ 2/3 & 1 & 0 \\ 10/3 & -2/5 & 1 \end{bmatrix} \begin{bmatrix} 3 & 1 & 4 \\ 0 & 10/3 & -5/3 \\ 0 & 0 & -10 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & 1 & 4 \\ 2 & 4 & 1 \\ 10 & 2 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & \textcircled{10/3} & -5/3 \\ 0 & -4/3 & -28/3 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 10/3 & -5/3 \\ 0 & 0 & -10 \end{bmatrix} = U$$

7. Consider $A\bar{x} = \bar{b}$, where $A = \begin{bmatrix} 1 & -3 & 0 & -1 \\ 5 & -17 & 3 & -10 \\ -3 & -7 & 26 & -35 \\ 7 & -27 & 5 & -27 \end{bmatrix}$ & $\bar{b} = \begin{bmatrix} -1 \\ 1 \\ 51 \\ 12 \end{bmatrix}$

Solve for $\bar{x}$ using the LU factorization of A .

$$\underline{U}: \begin{bmatrix} 1 & -3 & 0 & -1 \\ 5 & -17 & 3 & -10 \\ -3 & -7 & 26 & -35 \\ 7 & -27 & 5 & -27 \end{bmatrix}$$

$$\underline{L}: \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ -3 & 8 & 1 & 0 \\ 7 & 3 & -2 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 & -1 \\ 0 & -2 & 3 & -5 \\ 0 & -16 & 26 & -38 \\ 0 & -6 & 5 & -20 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 & -1 \\ 0 & -2 & 3 & -5 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & -4 & -5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -3 & 0 & -1 \\ 0 & -2 & 3 & -5 \\ 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & -1 \end{bmatrix} = U$$

So $A\bar{x} = \bar{b}$
means $LU\bar{x} = \bar{b}$

$$L\bar{y} = \bar{b}: \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 5 & 1 & 0 & 0 & 1 \\ -3 & 8 & 1 & 0 & 51 \\ 7 & 3 & -2 & 1 & 12 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 8 & 1 & 0 & 48 \\ 0 & 3 & -2 & 1 & 19 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 6 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix} \Rightarrow \bar{y} = \begin{bmatrix} -1 \\ 6 \\ 0 \\ 1 \end{bmatrix}$$

$$U\bar{x} = \bar{y}: \begin{bmatrix} 1 & -3 & 0 & -1 & -1 \\ 0 & -2 & 3 & -5 & 6 \\ 0 & 0 & 2 & 2 & 0 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 0 & -2 \\ 0 & -2 & 3 & 0 & 1 \\ 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 0 & 0 & -2 \\ 0 & -2 & 0 & 0 & -2 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & -1 \end{bmatrix} \Rightarrow \bar{x} = \begin{bmatrix} 1 \\ 1 \\ 1 \\ -1 \end{bmatrix}$$

8. find the determinants of the following matrices, using cofactor expansion or the definition of the determinant.

a) $\begin{bmatrix} 2 & 4 \\ 7 & -1 \end{bmatrix} \quad \det \begin{pmatrix} 2 & 4 \\ 7 & -1 \end{pmatrix} = -2 - 28 = -30$

b) $\begin{bmatrix} 1 & -3 & 4 \\ 7 & 2 & -2 \\ 4 & -5 & 1 \end{bmatrix} \Rightarrow \det = (2 + 24 - 140) - (32 + 10 - 21)$
 $= -114 - 21$
 $= -135$

c) $\det \begin{bmatrix} 4 & 1 & 0 & 2 & -5 \\ -2 & 3 & 0 & 4 & 7 \\ 4 & 0 & 3 & 2 & 1 \\ 7 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & -4 \end{bmatrix} = 3 \cdot \begin{vmatrix} 4 & 1 & 2 & -5 \\ -2 & 3 & 4 & 7 \\ 1 & 0 & 2 & -4 \end{vmatrix} = 3 \cdot 7 \cdot \begin{vmatrix} 1 & 2 & -5 \\ 3 & 4 & 7 \\ 0 & 2 & -4 \end{vmatrix}$
 $= 21 \left[1 \cdot \begin{vmatrix} 4 & 7 \\ 2 & -4 \end{vmatrix} - 3 \cdot \begin{vmatrix} 2 & -5 \\ 2 & -4 \end{vmatrix} \right] = 21 \left[(-16 - 14) - 3(-8 + 10) \right]$
 $= 21 \left[-30 - 3(2) \right] = 21 \cdot -36 = -756$

9. find the determinants of the following matrices, using properties of determinants. (hint look at #8)

a) $\begin{bmatrix} 1 & 7 & 4 \\ -3 & 2 & -5 \\ 4 & -2 & 1 \end{bmatrix} \overset{\text{transpose}}{=} \begin{bmatrix} 1 & -3 & 4 \\ 7 & 2 & -2 \\ 4 & -5 & 1 \end{bmatrix} = -135$

b) $\begin{bmatrix} 5 & -15 & 20 \\ 7 & 2 & -2 \\ 8 & -10 & 2 \end{bmatrix} \begin{vmatrix} 5 & -15 & 20 \\ 7 & 2 & -2 \\ 8 & -10 & 2 \end{vmatrix} = 5 \begin{vmatrix} 1 & -3 & 4 \\ 7 & 2 & -2 \\ 8 & -10 & 2 \end{vmatrix} = 5 \cdot 2 \begin{vmatrix} 1 & -3 & 4 \\ 7 & 2 & -2 \\ 4 & -5 & 1 \end{vmatrix}$
 $= -1350$

10. find the determinant, using row ~~operation~~ operations:

$\begin{vmatrix} 1 & 0 & 3 & -4 \\ 2 & 1 & 4 & 1 \\ -2 & -5 & 2 & 0 \\ 3 & 2 & 1 & -1 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 9 \\ 0 & -5 & 8 & -8 \\ 0 & 2 & -8 & 11 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 9 \\ 0 & 0 & -2 & 37 \\ 0 & 0 & -4 & -7 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 3 & -4 \\ 0 & 1 & -2 & 9 \\ 0 & 0 & -2 & 37 \\ 0 & 0 & 0 & -81 \end{vmatrix}$
 $= 1 \cdot 1 \cdot (-2) \cdot (-81)$
 $= 162$

11. Solve the following system using row operations, and write your answer in parametric vector form:

$$\begin{cases} x + 11y + 2z + w = 9 \\ 2x - 3y - z - 6w = +16 \\ 4x + y - 3z + 5w = 25 \end{cases}$$

$$\begin{bmatrix} 1 & 11 & 2 & 1 & 9 \\ 2 & -3 & -1 & -6 & 16 \\ 4 & 1 & -3 & 5 & 25 \end{bmatrix} \quad \begin{array}{l} R_2 = R_2 - 2R_1 \\ R_3 = R_3 - 4R_1 \end{array}$$

$$\begin{bmatrix} 1 & 11 & 2 & 1 & 9 \\ 0 & -25 & -5 & -8 & -2 \\ 0 & -43 & -11 & 1 & -11 \end{bmatrix} \quad -\frac{1}{25}R_2$$

$$\begin{bmatrix} 1 & 11 & 2 & 1 & 9 \\ 0 & 1 & 1/5 & 8/25 & 2/25 \\ 0 & -43 & -11 & 1 & -11 \end{bmatrix} \quad \begin{array}{l} R_1 = R_1 - 11R_2 \\ R_3 = R_3 + 43R_2 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1/5 & -63/25 & 203/25 \\ 0 & 1 & 1/5 & 8/25 & 2/25 \\ 0 & 0 & -12/5 & 369/25 & -189/25 \end{bmatrix} \quad R_3 = -\frac{5}{12}R_3$$

$$\begin{bmatrix} 1 & 0 & 0 & -15/4 & 35/4 \\ 0 & 1 & 0 & 3/20 & -11/20 \\ 0 & 0 & 1 & -123/20 & 63/20 \end{bmatrix}$$

$$\Rightarrow \quad \begin{aligned} x &= \frac{15}{4}w + \frac{35}{4} \\ y &= -\frac{31}{20}w - \frac{11}{20} \\ z &= \frac{123}{20}w + \frac{63}{20} \end{aligned}$$

So

$$\text{Ans} = \begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} = \begin{bmatrix} 15/4 \\ -31/20 \\ 123/20 \\ 1 \end{bmatrix} w + \begin{bmatrix} 35/4 \\ -11/20 \\ 63/20 \\ 0 \end{bmatrix}$$

12. Determine which of the following sets are linearly independent or linearly dependent.

- a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$ Lin. Indep. (1 nonzero vector)
- b) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \end{bmatrix} \right\}$ Lin. Indep. ($\bar{u} \neq k\bar{v} \forall k$)
- c) $\left\{ \begin{bmatrix} 1 \\ 4 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$ Lin. Depend. (zero vector)
- d) $\left\{ \begin{bmatrix} -1 \\ 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 5 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \\ 1 \end{bmatrix} \right\}$ Lin. Indep. (pivot in every column)
- e) $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \end{bmatrix} \right\}$ Lin. Depend. (3 vector in $\mathbb{R}^2$)

13. Which of the following are linear transformations?

- a) $T(x_1, x_2, x_3) = (2x_1 - 3x_2 + x_3, x_1 - 4x_3, 2x_2)$ yes
- (i) $T(\bar{x}) + T(\bar{y}) = T(x_1, x_2, x_3) + T(y_1, y_2, y_3) = (2x_1 - 3x_2 + x_3, x_1 - 4x_3, 2x_2) + (2y_1 - 3y_2 + y_3, y_1 - 4y_3, 2y_2)$
 $= (2(x_1 + y_1) - 3(x_2 + y_2) + (x_3 + y_3), (x_1 + y_1) - 4(x_3 + y_3), 2(x_2 + y_2)) = T(\bar{x} + \bar{y})$
- (ii) $T(c\bar{x}) = T(cx_1, cx_2, cx_3) = (2cx_1 - 3cx_2 + cx_3, cx_1 - 4cx_3, 2cx_2)$
 $= c(2x_1 - 3x_2 + x_3, x_1 - 4x_3, 2x_2) = cT(\bar{x})$
- b) $T(x_1, x_2) = (x_1 + 3x_2, x_1 x_2, 3x_2)$ no
- $T(2\bar{x}) = T(2x_1, 2x_2) = (2x_1 + 6x_2, 4x_1 x_2, 6x_2) = 2(x_1 + 3x_2, 2x_1 x_2, 3x_2)$
 $\uparrow$
 So $T(2\bar{x}) \neq 2T(\bar{x})$
- c) $T(x_1, x_2, x_3, x_4) = (3x_2 - x_4, x_3 + x_4 + 1, -5x_1)$ no
- $T(0, 0, 0, 0) = (0, 1, 0)$
 So $T(\bar{0}) \neq \bar{0}$

14. For each mapping in #13, if it's a linear transformation, find its standard matrix and inverse transformation.

Only 13(a) is a linear transformation, so $T(\bar{x}) = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & -4 \\ 0 & 2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$.

The standard matrix is $A = \begin{bmatrix} 2 & -3 & 1 \\ 1 & 0 & -4 \\ 0 & 2 & 0 \end{bmatrix}$.

Then $A^{-1} = \begin{bmatrix} 4/9 & 1/9 & 2/3 \\ 0 & 0 & 1/2 \\ 1/9 & -2/9 & 1/6 \end{bmatrix}$, and $T^{-1}(\bar{x}) = A^{-1}\bar{x}$.

15. The following matrices are the std matrices for a Linear transformation. Determine the domain & codomain of that transformation as well as whether the transformation is 1-to-1 or onto.

a) $\begin{bmatrix} 2 & -1 \\ 3 & 0 \\ 1 & 5 \end{bmatrix}$

$T: \mathbb{R}^2 \rightarrow \mathbb{R}^3$

not onto (can't span $\mathbb{R}^3$ - too narrow)
is 1-1 (b/c columns lin. indep.)

b) $\begin{bmatrix} 2 & -1 & 3 & 1 \\ 4 & 0 & 2 & 1 \end{bmatrix}$

$T: \mathbb{R}^4 \rightarrow \mathbb{R}^2$

not 1-1 (too wide)
is onto (spans $\mathbb{R}^2$) (pivot in each row)

c) $\begin{bmatrix} 4 & 0 & 2 \\ 5 & 3 & 2 \\ -1 & 7 & -8 \end{bmatrix}$

$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

is 1-1 (columns lin. indep.)
is onto (spans $\mathbb{R}^3$)
(ie: pivot in every row/col)

d) $\begin{bmatrix} 2 & -3 & 0 \\ -1 & 4 & 5 \\ -3 & 1 & -7 \end{bmatrix}$

$T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

not 1-1 (only 2 pivots)
not onto