Homework Set 2

(sections 1.1 – 1.4)

Determine whether the following systems are consistent and/or unique.

1. $\begin{cases} x_1 + 2x_2 + x_3 = 4 \\ x_2 - x_3 = 1 \\ x_1 + 3x_2 = 0 \end{cases}$

2. $\begin{cases} x_1 - x_4 = 3 \\ 2x_2 + 2x_3 = 0 \\ x_3 + 3x_4 = 2 \\ -2x_1 + 3x_2 + 2x_3 + x_4 = 5 \end{cases}$

Determine the value(s) of *h* and/or *k* such that the matrix is the augmented matrix of a consistent system.

3. $\begin{bmatrix} 1 & h & 3 \\ -2 & 4 & 8 \end{bmatrix}$

4. $\begin{bmatrix} 6 & -8 & 5 \\ 3 & -4 & h \end{bmatrix}$

5. $\begin{bmatrix} 3 & -1 & h \\ -12 & 4 & k \end{bmatrix}$

6. $\begin{bmatrix} 2 & -1 & 3 & h \\ 1 & 0 & 4 & k \\ -6 & 3 & -9 & 15 \end{bmatrix}$

Find the reduced row echelon form of the following matrices. Interpret your result by giving the solutions of the systems whose augmented matrix is the one given.

7. $\begin{bmatrix} 4 & 3 & 0 & 7 \\ 8 & 6 & 2 & -3 \end{bmatrix}$

8. $\begin{bmatrix} 1 & 6 & -3 & -5 \\ 7 & 1 & 1 & 4 \\ 0 & 4 & -1 & 3 \end{bmatrix}$

9. $\begin{bmatrix} 0 & 4 & 7 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 3 & 1 & -4 \end{bmatrix}$

10. $\begin{bmatrix} 0 & 0 & 3 & -1 & 5 \\ 1 & 0 & 0 & 4 & 2 \\ 4 & 1 & 3 & 0 & -8 \\ 1 & 2 & 7 & 9 & 0 \end{bmatrix}$

Answer the following theory questions as concisely as possible.

11. Suppose a 3×5 *coefficient* matrix for a system has three pivot columns. Is the system consistent? Why or why not?

12. Suppose a system of linear equations has a 3×5 *augmented* matrix whose fifth column is a pivot column. Is the system consistent? Why or why not?

13. Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

14. Suppose the coefficient matrix of a linear system of three equations in three variables has a pivot in each column. Explain why the system has a unique solution.

15. A system of linear equations with fewer equations then unknowns is called an *underdetermined system*. Suppose that such a system happens to be consistent. Explain why there must be infinitely many solutions to the system.

16. A system of linear equations with more equations then unknowns is called an *overdetermined system*. Can such a system be consistent? Illustrate your answer with a specific example of a system with three equations in two unknowns.

Given the vectors **u** and **v**, find **u**+**v**, 2**u**-3**v**, -**u**, and **v**-**u**

17. $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

18. $\mathbf{u} = \begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 5 \\ -1 \end{bmatrix}$

Determine whether the vector **b** is in the span of $\boldsymbol{a_1}$, $\boldsymbol{a_2}$, and $\boldsymbol{a_3}$

19. $a_1 = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix}$, $a_2 = \begin{bmatrix} 0 \\ 0 \\ 5 \end{bmatrix}$, $a_3 = \begin{bmatrix} 3 \\ 6 \\ 1 \end{bmatrix}$; $b = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

20. $a_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $a_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$, $a_3 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$; $b = \begin{bmatrix} -5 \\ 17 \\ 6 \end{bmatrix}$

21. Let A = $\begin{bmatrix} 2 & 0 & 6 \\ -1 & 8 & 5 \\ 1 & -2 & 1 \end{bmatrix}$ and **b** = $\begin{bmatrix} 10 \\ 3 \\ 3 \end{bmatrix}$. Let W be the set of all linear combinations of the columns of A.

a. Is b in W?

b. Show that the third column of A is in W.

Determine whether the given vectors span $\mathbb{R}^4$. Explain why or why not.

22. $\begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ -1 \end{bmatrix}$

23. $\begin{bmatrix} 1 \\ 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 2 \\ -8 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ -5 \\ 7 \\ -1 \end{bmatrix}$

Compute the following matrix-vector products.

24. $\begin{bmatrix} 4 & 2 & 8 \\ 0 & -1 & 1 \\ 2 & 7 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 2 \end{bmatrix} =$

25. $\begin{bmatrix} 1 & 4 & 7 \\ 8 & 3 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \end{bmatrix} =$