Homework Set 5

(sect 2.3 – 2.5)

1. Determine if the following matrix is invertible. Use as few calculations as possible.

$$\begin{bmatrix} 11 & 1 & 0 \\ 7 & 3 & 0 \\ 4 & -2 & 0 \end{bmatrix}$$

2. Can a square matrix with two identical columns or with two identical rows be invertible? Why or why not?

3. If the columns of a 10×10 matrix A are linearly independent, what do you know about the solution set of $A\boldsymbol{x} = \boldsymbol{b}$? Why?

4. Consider the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ given by $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 4x_1 - 5x_2 \\ 7x_1 + 3x_2 \end{bmatrix}$. Show that T is invertible, and find a formula for the inverse transformation T^{-1}

5. You want to compute $A \cdot B$

$$A = \left[\begin{array}{cc:ccc} 1 & -1 & -2 & -3 & 0 \\ 1 & 0 & 5 & 3 & 2 \\ 5 & 3 & 1 & 0 & 4 \end{array}\right] \qquad B = \begin{bmatrix} 3 & 0 & 1 & -1 \\ -2 & 1 & 0 & 0 \\ 0 & 5 & -3 & 3 \\ 9 & 2 & 0 & 4 \\ 3 & 0 & -1 & 0 \end{bmatrix}$$

a. A partition for A is given. Give a partition for B such that A and B can be multiplied together as block matrices. (draw your partition on matrix B above)

b. Compute $A \cdot B$ using submatrices.

6. a. Show that $A^2 = I$ where $A = \begin{bmatrix} 1 & 0 \\ 3 & -1 \end{bmatrix}$

b. Use partitions to make M a block matrix in order to show that $M^2 = I$ where

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ 0 & 1 & -3 & 1 \end{bmatrix}$$

c. Use the idea of part (a), to construct a 5×5 matrix $M = \begin{bmatrix} A & 0 \\ C & D \end{bmatrix}$ such that $M^2 = I$. Make C a nonzero 2×3 matrix. Show that your construction works by computing M^2.

7. Verify that the given L and U give the LU factorization of the matrix A.

$$A = \begin{bmatrix} 1 & -2 & 4 & 5 \\ 5 & -6 & 21 & 29 \\ -2 & 8 & -9 & -4 \\ 3 & -10 & 3 & 16 \end{bmatrix}, \quad L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 5 & 1 & 0 & 0 \\ -2 & 1 & 1 & 0 \\ 3 & -1 & 4 & 1 \end{bmatrix}, \text{ and } U = \begin{bmatrix} 1 & -2 & 4 & 5 \\ 0 & 4 & 1 & 4 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 0 & -3 \end{bmatrix}$$

8. Find the LU factorization of the given matrix A

$$A = \begin{bmatrix} 2 & 7 & -5 & 2 & 1 \\ 2 & 10 & -7 & 5 & 5 \\ -2 & 2 & 6 & 8 & 16 \\ 4 & 14 & 4 & 6 & 11 \end{bmatrix}$$

9. Solve the equation $A\boldsymbol{x} = \boldsymbol{b}$ by using LU factorization.

$$A = \begin{bmatrix} 3 & -4 & 2 \\ 3 & -2 & 1 \\ 15 & -16 & 15 \end{bmatrix}, \text{ and } \boldsymbol{b} = \begin{bmatrix} 1 \\ 2 \\ 14 \end{bmatrix}$$

10. Solve the equation $A\boldsymbol{x} = \boldsymbol{b}$ by using LU factorization. (hint: let $\ell_{54} = 0$)

$$A = \begin{bmatrix} 4 & -5 & 6 \\ 8 & -7 & 16 \\ 20 & 5 & 83 \\ 4 & 1 & 79 \\ -8 & 7 & 23 \end{bmatrix}, \text{ and } \boldsymbol{b} = \begin{bmatrix} -14 \\ -22 \\ -10 \\ -2 \\ 22 \end{bmatrix}$$