

Sect 1.4
Answer key

1. can't compute

2. can't compute

3. $\begin{bmatrix} -3 \\ 1 \\ 4 \end{bmatrix}$

4. $\begin{bmatrix} 7 \\ 8 \end{bmatrix}$

5. $5 \begin{bmatrix} 5 \\ -2 \end{bmatrix} - 1 \cdot \begin{bmatrix} 1 \\ -7 \end{bmatrix} + 3 \begin{bmatrix} -8 \\ 3 \end{bmatrix} - 2 \begin{bmatrix} 4 \\ -5 \end{bmatrix} = \begin{bmatrix} -8 \\ 16 \end{bmatrix}$

6. $-2 \begin{bmatrix} 7 \\ 2 \\ 9 \\ -3 \end{bmatrix} - 5 \begin{bmatrix} -3 \\ 1 \\ -6 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ -9 \\ 12 \\ -4 \end{bmatrix}$

7. $\begin{bmatrix} 4 & -5 & 7 \\ -1 & 3 & -8 \\ 7 & -5 & 0 \\ -4 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -8 \\ 0 \\ -7 \end{bmatrix}$

8. $\begin{bmatrix} 4 & -4 & -5 & 3 \\ -2 & 5 & 4 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$

9. $x_1 \begin{bmatrix} 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -5 \\ 4 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

10. $x_1 \begin{bmatrix} 8 \\ 5 \\ 1 \end{bmatrix} + x_2 \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

and
 $\begin{bmatrix} 3 & 1 & -5 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 9 \\ 0 \end{bmatrix}$

and
 $\begin{bmatrix} 8 & -1 \\ 5 & 4 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 2 & 4 & -2 \\ 0 & 1 & 5 & 2 \\ -2 & -4 & -3 & 9 \end{bmatrix}, \bar{x} = \begin{bmatrix} 0 \\ -3 \\ 1 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 2 & 1 & 0 \\ -3 & -1 & 2 & 1 \\ 0 & 5 & 3 & -1 \end{bmatrix}, \bar{x} = \begin{bmatrix} 3/5 \\ -4/5 \\ 1 \end{bmatrix}$

13. yes

15. no solution when
 $3b_1 + b_2 \neq 0$

solution consistent when
 $b_2 = -3b_1$

19. No \neq columns of A
do not span \mathbb{R}^4

17. 3 rows contain a pivot
no solution for all b in \mathbb{R}^4

25. $c_1 = -3, c_2 = -1, c_3 = 2$

23. a) false
b) true
c) false
d) true
e) true
f) true
24. a) true
b) true
c) true
d) true
e) false
f) true

26. $3u - 5v - w = 0 \Rightarrow w = 3u - 5v$
so 1 solution is $x_1 = 3, x_2 = -5$

31.

$$A = \begin{bmatrix} * & * \\ * & * \\ * & * \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad b = \begin{bmatrix} * \\ * \\ * \end{bmatrix}$$

$$\Rightarrow [A \ b] = \begin{bmatrix} * & * & * \\ * & * & * \\ * & * & * \end{bmatrix}$$

will only be consistent if the last (or 1 of equations) equation is a linear combination

So, there exists a \bar{b} such that

$$\text{rref}[A \ b] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \leftarrow \text{inconsistent}$$

32. 3 vectors can't span \mathbb{R}^4 since $3 < 4$

n vectors can't span \mathbb{R}^m if $n < m$