Sect 1.9
Answer key

1. $\begin{bmatrix} 3 & -5 \\ 1 & 2 \\ 3 & 0 \\ 1 & 0 \end{bmatrix}$

2. $\begin{bmatrix} 1 & 4 & -5 \\ 3 & -7 & 4 \end{bmatrix}$

3. $\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

4. $\begin{bmatrix} 1/\sqrt{2} & 1/\sqrt{2} \\ -1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

5. $\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$

6. $\begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}$

7. $\begin{bmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$

8. $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

9. $\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix}$

10. $\begin{bmatrix} 0 & -1 \\ -1 & 0 \end{bmatrix}$

11. T takes $e_1 \to -e_1$
& $e_2 \to -e_2$
a rotation thru π rad (180°) does the same; hence, these are really the same transformation (ie: the std matrices of the 2 are identical)

15. $\begin{bmatrix} 3 & 0 & -2 \\ 4 & 0 & 0 \\ 1 & -1 & +1 \end{bmatrix}$

16. $\begin{bmatrix} 1 & -1 \\ -2 & 1 \\ 1 & 0 \end{bmatrix}$

17. $\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

18. $\begin{bmatrix} -3 & 2 \\ 1 & -4 \\ 0 & 0 \\ 0 & 1 \end{bmatrix}$

19. $\begin{bmatrix} 1 & -5 & 4 \\ 0 & 1 & -6 \end{bmatrix}$

20. $\begin{bmatrix} 2 & 0 & 3 & -4 \end{bmatrix}$

23. a) T
b) T
c) F
d) F
e) F

24. a) F
b) T
c) T
d) F
e) T

25. not 1-1
not onto

26. not 1-1
is onto

27. not 1-1
is onto

28. $T(x) = Ax$. A has Lin Indep columns since $\bar{a}_1$ & $\bar{a}_2$ not multiples $\Rightarrow$ T is 1-1
A has pivot in each column b/c $Ax=0$ has no free var.
$\Rightarrow$ columns span $\mathbb{R}^2$ $\Rightarrow$ T is onto

31. n

32. A has m pivot columns
iff A has pivot in each row
iff columns of A span $\mathbb{R}^m$
iff T is onto

33. A is standard matrix & $T(\bar{x}) = B\bar{x}$
So $A\bar{x} = T(\bar{x}) = B\bar{x}$, want $A = B$
Let e_j be j^{th} column of I
$T(e_j) = Be_j = \underbrace{b_j}_{\text{def}} = j^{th}$ column of B
but $T(e_j) = j^{th}$ column of A by def
$\Rightarrow$ $\bar{a}_j = \bar{b}_j$ for all j
$\Rightarrow$ $A = B$

34. T onto iff $\exists x \in \mathbb{R}^n \ni \bar{y} = T(\bar{x})$

35. onto: $n \le m$
1-1: $n \ge m$
$\Big\}$ both: $n = m$