

Sect 2.4
Answer key

4.
$$\begin{bmatrix} A & B \\ -XA+C & -XB+D \end{bmatrix}$$

5.
$$Y = B^{-1}$$

$$X = -B^{-1}A, Z = C$$

6. Assume $A, X, C, \& Z$ square
then $X = A^{-1} \& Z = C^{-1}$
 $\Rightarrow Y = -C^{-1}BA^{-1}$

10.
$$X = -A + BC$$

$$Y = -B$$

$$Z = -C$$

13. (a) Suppose A^{-1} exists

Let
$$A^{-1} = \begin{bmatrix} D & E \\ F & G \end{bmatrix}$$

~~Suppose~~

then
$$AA^{-1} = \begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} D & E \\ F & G \end{bmatrix} = \begin{bmatrix} BD & BE \\ CF & CG \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix}$$

So $BD = I \& CG = I$

means $B \& C$ are invertible

(b) Suppose $B^{-1} \& C^{-1}$ exist

$$\begin{bmatrix} B & 0 \\ 0 & C \end{bmatrix} \begin{bmatrix} B^{-1} & 0 \\ 0 & C^{-1} \end{bmatrix} = \begin{bmatrix} BB^{-1} & 0 \\ 0 & CC^{-1} \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & I \end{bmatrix} = I$$

So A^{-1} exists

15
$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} I & 0 \\ A_{21}A_{11}^{-1} & I \end{bmatrix} \begin{bmatrix} A_{11} & 0 \\ 0 & S \end{bmatrix} \begin{bmatrix} I & A_{11}^{-1}A_{12} \\ 0 & I \end{bmatrix}$$