Sect 2.5
Answer key

1. $L\bar{y} = \bar{b} \Rightarrow \bar{y} = \begin{bmatrix} -7 \\ -2 \\ 6 \end{bmatrix}$

$U\bar{x} = \bar{y} \Rightarrow \bar{x} = \begin{bmatrix} 3 \\ 4 \\ -6 \end{bmatrix}$

2. $L\bar{y} = \bar{b} \Rightarrow \bar{y} = \begin{bmatrix} 2 \\ -2 \\ 2 \end{bmatrix}$

$U\bar{x} = \bar{y} \Rightarrow \bar{x} = \begin{bmatrix} 1/4 \\ 2 \\ 1 \end{bmatrix}$

7. $LU = \begin{bmatrix} 1 & 0 \\ -3/2 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 5 \\ 0 & 7/2 \end{bmatrix}$

8. $LU = \begin{bmatrix} 1 & 0 \\ 2/3 & 1 \end{bmatrix} \begin{bmatrix} 6 & 9 \\ 0 & -1 \end{bmatrix}$

9. $\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 3 & 2/3 & 1 \end{bmatrix} \begin{bmatrix} 3 & -1 & 2 \\ 0 & -3 & 12 \\ 0 & 0 & -8 \end{bmatrix}$

10. $\begin{bmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ -3 & -5 & 1 \end{bmatrix} \begin{bmatrix} -5 & 3 & 4 \\ 0 & -2 & -1 \\ 0 & 0 & 9 \end{bmatrix}$

11. $\begin{bmatrix} 1 & 0 & 0 \\ 2 & 1 & 0 \\ -1/3 & 10 & 1 \end{bmatrix} \begin{bmatrix} 3 & -6 & 3 \\ 0 & 5 & -4 \\ 0 & 0 & 5 \end{bmatrix}$

12. $\begin{bmatrix} 1 & 0 & 0 \\ 1/2 & 1 & 0 \\ -3 & -2 & 1 \end{bmatrix} \begin{bmatrix} 2 & -4 & 2 \\ 0 & 7 & -5 \\ 0 & 0 & 0 \end{bmatrix}$

13. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 4 & 5 & 1 & 0 \\ -2 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 & -5 & -3 \\ 0 & -2 & 3 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

14. $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 3 & 1 & 0 & 0 \\ -2 & -1 & 1 & 0 \\ -1 & -2 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 4 & -1 & 5 \\ 0 & -5 & 1 & -6 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

17. $U^{-1} = \begin{bmatrix} 1/4 & 3/8 & 1/4 \\ 0 & -1/2 & 1/2 \\ 0 & 0 & 1/2 \end{bmatrix}$

$L^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1/8 & 3/8 & 1/4 \\ -3/2 & -1/2 & 1/2 \\ -1 & 0 & 1/2 \end{bmatrix}$

24. Q square $\Rightarrow Q^TQ = I$ & $Q^{-1} = Q^T$

So A is invertible

$\Rightarrow A\bar{x} = \bar{b}$ has a unique solution

so
$$QR\bar{x} = \bar{b}$$
$$Q^TQR\bar{x} = Q^Tb$$
$$R\bar{x} = Q^Tb$$
$$\Rightarrow \bar{x} = R^{-1}Q^Tb$$

25. $U, D,$ & V^T are invertible

$$\Rightarrow A^{-1} = (UDV^T)^{-1}$$
$$= (V^T)^{-1}D^{-1}U^{-1}$$
$$= VDU^T$$

26. in general, $A^k = P D^k P^{-1}$

where $D^k = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1/2^k & 0 \\ 0 & 0 & 1/3^k \end{bmatrix}$

31. (a)

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1/4 & -.0667 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -.2667 & -.2857 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -.2679 & -.0833 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -.2917 & -.2921 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -.2697 & -.0861 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -.2948 & -.2931 & 1 \end{bmatrix}$$

$$U = \begin{bmatrix} 4 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3.75 & -.25 & -1 & 0 & 0 & 0 & 0 \\ 0 & & 3.7333 & -1.0667 & -1 & 0 & 0 & 0 \\ 0 & & & 3.4286 & -.2857 & -1 & 0 & 0 \\ 0 & & & & 3.7083 & -1.0833 & -1 & 0 \\ 0 & & \text{all zeros} & & & 3.3919 & -.2921 & -1 \\ 0 & & & & & & 3.7052 & -1.0861 \\ 0 & & & & & & & 3.3868 \end{bmatrix}$$

(b) $\bar{x} = (3.9569, 6.5885, 4.2392, 7.3971, 5.6029, 8.7608, 9.4115, 12.0431)$

(c)

$$A^{-1} = \begin{bmatrix} .2953 & .0866 & & & & & & \\ .0866 & .2953 & & & & & & \\ .0945 & .0509 & .3271 & & & & & \\ .0509 & .0945 & .1093 & .3271 & & & & \\ .0318 & .0227 & .1045 & .0591 & .3271 & & & \\ .0227 & .0318 & .0591 & .1045 & .1093 & .3271 & & \\ .01 & .0082 & .0318 & .0227 & .0945 & .0509 & .2953 & \\ .0082 & .1 & .0227 & .0318 & .0509 & .0945 & .0866 & .2953 \end{bmatrix}$$

Symmetric $\Rightarrow (A^{-1})^T = (A^{-1})$