# Project 2

## Section 1.6

4. a) exchange table

| Agriculture | Energy | Manufacturing | Transp. | Purchased by |
|---|---|---|---|---|
| .65 | .30 | .30 | .20 | Agriculture |
| .10 | .10 | .15 | .10 | Energy |
| .25 | .35 | .15 | .30 | Manufacturing |
| 0 | .25 | .40 | .40 | Transportation |

Outputs → Inputs

b) Equilibrium Prices

price Agriculture $= p_A = .65p_A + .30p_E + .30p_m + .20p_T$

price Energy $= p_E = .10p_A + .10p_E + .15p_m + .10p_T$

price Manufacturing $= p_m = .25p_A + .35p_E + .15p_m + .30p_T$

price Transporta $= p_T = 0 + .25p_E + .40p_m + .40p_T$

$$\Rightarrow \begin{cases} -.35p_A + .3p_E + .3p_m + .2p_T = 0 \\ .10p_A + -.9p_E + .15p_m + .1p_T = 0 \\ .25p_A + .35p_E - .85p_m + .3p_T = 0 \\ .25p_E + .4p_m - .6p_T = 0 \end{cases}$$

$$\left[\begin{array}{cccc|c} -.35 & .3 & .3 & .2 & 0 \\ .1 & -.9 & .15 & .1 & 0 \\ .25 & .35 & -.85 & .3 & 0 \\ 0 & .25 & .4 & -.6 & 0 \end{array}\right]$$

after we put into rref we get

$$\longrightarrow \begin{bmatrix} 1 & 0 & 0 & -2.03 & 0 \\ 0 & 1 & 0 & -.53 & 0 \\ 0 & 0 & 1 & -1.17 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

rounded to 2 decimal places

$$\Rightarrow \quad \bar{p} = \begin{bmatrix} p_A \\ p_E \\ p_m \\ p_T \end{bmatrix} = \begin{bmatrix} 2.03 \\ .53 \\ 1.17 \\ 1 \end{bmatrix} p_T$$

So one solution is

$$p_A = 203, \quad p_E = 53, \quad p_m = 117, \quad p_T = 100$$

12. a) general traffic pattern

| Intersect | Flow in | Flow out |
|---|---|---|
| A | $x_1$ | $40 + x_3 + x_4$ |
| B | 200 | $x_1 + x_2$ |
| C | $x_2 + x_3$ | $100 + x_5$ |
| D | $x_4 + x_5$ | 60 |

$$\Rightarrow \begin{cases} x_1 = 40 + x_3 + x_4 \\ 200 = x_1 + x_2 \\ x_2 + x_3 = 100 + x_5 \\ x_4 + x_5 = 60 \end{cases}$$

$$\left[\begin{array}{ccccc|c} 1 & 0 & -1 & -1 & 0 & 40 \\ 1 & 1 & 0 & 0 & 0 & 200 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \end{array}\right]$$

now we row reduce to get

$$\longrightarrow \left[\begin{array}{ccccc|c} 1 & 0 & -1 & 0 & 1 & 100 \\ 0 & 1 & 1 & 0 & -1 & 100 \\ 0 & 0 & 0 & 1 & 1 & 60 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

so $x_3$ & $x_5$ are free

$$\Rightarrow \begin{cases} x_1 = x_3 - x_5 + 100 \\ x_2 = -x_3 + x_5 + 100 \\ x_3 \text{ free} \\ x_4 = -x_5 + 60 \\ x_5 \text{ free} \end{cases}$$

b) general flow when $x_4$ is closed

this means $x_4 = 0 \Rightarrow x_5 = 60$

and plugging in for the rest of the equations we get:

$$\begin{cases} x_1 = x_3 + 40 \\ x_2 = 160 - x_3 \\ x_3 \text{ free} \\ x_4 = 0 \\ x_5 = 60 \end{cases}$$

c) min value of $x_1$ when $x_4 = 0$

we know $x_1 = x_3 + 40$

and all the $x_i$'s are non-negative

So min $x_1 = 0 + 40 = \boxed{40}$ (when $x_3 = 0$)

14. general flow & min $X_6$

| intersection | flow in | flow out |
|---|---|---|
| A | $X_1$ | $X_2+100$ |
| B | $X_2+50$ | $X_3$ |
| C | $X_3$ | $X_4+120$ |
| D | $X_4+150$ | $X_5$ |
| E | $X_5$ | $X_6+80$ |
| F | $X_6+100$ | $X_1$ |

$$\Rightarrow \begin{cases} X_1 = X_2+100 \\ X_2 + 50 = X_3 \\ X_3 = X_4+120 \\ X_4+150 = X_5 \\ X_5 = X_6+80 \\ X_6+100 = X_1 \end{cases}$$

$$\Rightarrow \begin{cases} X_1 = X_6+100 \\ X_2 = X_6 \\ X_3 = X_6+50 \\ X_4 = X_6-70 \\ X_5 = X_6+80 \\ X_6 \text{ free} \end{cases}$$

and all $X_i$'s must equal zero or larger, so smallest $X_6$ satisfies: $0 = X_6 - 70$

OR $\min X_6 = 70$