```
C = swap(C,1,2)
```

and repeat the operations. (You can complete this example in Exercise 8.) This time we get the system

$$\begin{bmatrix} 0 & 1-u & 1-2u \\ 1 & 1 & 2 \end{bmatrix}$$

Here $1-u$ and $1-2u$ are nearly the same. But this is harmless, as the system

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix}$$

has the acceptable solution. Choosing to pivot on the 1 instead of the u makes the difference. While both choices of pivot lead to a roundoff error, the choice of the larger value in the (2, 1) position means that the scalar m will be smaller and thus the arithmetic adjustments in the replacement will also be smaller.

The *partial pivoting strategy* is to *pivot on the largest entry (in absolute value) below the pivot entry and in the same column.* In our example, the two eligible entries in the first column are u and 1. The partial pivoting strategy chooses 1 since it is larger. MATLAB utilizes Gaussian Elimination with partial pivoting in the A\b function, which, as we have seen provides an accurate answer to the example system.

$x+y=x$ up until $y=\sqrt{2}\,10^{17}$

MATLAB Exercises

it holds 15 decimal places:
$(32-d)=15 \Rightarrow d=17$

1. Let

```
x = pi*10 ^ 32
y = sqrt(2)*10 ^ -101
```

Show that x + y and x are identical floating point numbers. Then progressively raise the exponent on y until you find a value for which x + y and x are different. Looking at the exponents on x and y, explain what happened here.

2. MATLAB can represent complex numbers easily. Look at sqrt(-1). THen use the quadratic formula to find the roots of x^2+x+1 with MATLAB. $-0.5000 + 0.8660i$
$-0.5000 - 0.8660i$

3. MATLAB has some important conventions for displaying matrices. Try the following:

```
A = rand(4)*10
1000*A
```

difference is shown above the entries

Do you get the same thing? The entries of A and A*1000 appear to be the same (in base 10), but the exponents are different. Where does the display show this difference in the exponents? Now try the following:

```
v = [1 2 ^ -53]
```

Is v(1) the same as 1? Is v(2) the same as 0?
yes
No, it's 1.1102×10^{-16}

4. Roundoff error will occur with very simple computations. Try the following:

```
1/5 == .2                       ✓
1/5 + 1/5 == .4                 ✓
1/5 + 1/5 + 1/5 == .6           ← not equal
1/5 + 1/5 + 1/5 + 1/5 == .8
```

When does MATLAB start to indicate that these expressions are not equal? As a general rule, you should not rely on the function ==. As we see here, small differences can emerge between values that are mathematically the same. A better check is to test for proximity with

```
abs(1/5 + 1/5 + 1/5 - .6) < 2
```

5. Look at the following:

`2 ^ 1023` $= 8.9885 \cdot 10^{307}$

and

`2 ^ 1024` = Inf

This is *overflow.* Try the following to see if `Inf` behaves in a reasonable way:

`Inf*2` = Inf

`Inf ^ 2` = Inf

`1/Inf` = 0

`Inf + 2` = Inf

`2 - Inf` = -Inf

`Inf*Inf` = Inf

`Inf + Inf` = Inf

6. Try these:

`Inf - Inf` = NaN

`Inf / Inf` = NaN

The indeterminate forms `Inf - Inf` and `Inf / Inf` are `NaN`, read as "not a number." Describe what happens with the following:

`NaN*2` = NaN

`NaN ^ 2` = NaN

`1/NaN` = NaN

`NaN + 2` = NaN

7. *Underflow* occurs when a computation leads to an exponent smaller than the minimum exponent allowed by the floating point arithmetic. Compare

`2 ^ -1100` = 0

with

`2 ^ -1000` $= 9.3326 \cdot 10^{-302}$

Find the largest exponent e for which 2^e causes underflow. -1075

8. Use the MATLAB commands `swap`, `scale`, and `replace` to find the solution to the example system used in the discussion.

9. Use the MATLAB commands `swap`, `scale`, and `replace` and partial pivoting to find the solution to the system `A = magic(3); b = [1;2;3]`. The partial pivoting strategy will require a row switch.

8. $\begin{bmatrix} 0 & 1 & 1 \\ 1 & 1 & 2 \end{bmatrix} = C$

$C = \text{swap}(C, 1, 2)$ $\qquad \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1 & 1 \end{bmatrix}$

$C = \text{replace}(C, 1, -1, 2)$ $\qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

So $x = 1$, $y = 1$

$\begin{bmatrix} 0 & 1-u & 1-2u \\ 1 & 1 & 2 \end{bmatrix} = C$

$C = \text{swap}(C, 1, 2)$ $\qquad \begin{bmatrix} 1 & 1 & 2 \\ 0 & 1-u & 1-2u \end{bmatrix}$

$C = \text{replace}\left(C, 1, \frac{-1}{1-u}, 2\right)$ $\qquad \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$

9. $A = \text{magic}(3)$, $b = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$

$C = [A, b]$ $\qquad \begin{bmatrix} 8 & 1 & 6 & 1 \\ 3 & 5 & 7 & 2 \\ 4 & 9 & 2 & 3 \end{bmatrix}$

$C = \text{scale}(C, 1, 1/8)$ $\qquad \begin{bmatrix} 1 & 1/8 & 3/4 & 1/8 \\ 3 & 5 & 7 & 2 \\ 4 & 9 & 2 & 3 \end{bmatrix}$

$C = \text{replace}(C, 2, -3, 1)$ $\qquad \begin{bmatrix} 1 & 1/8 & 3/4 & 1/8 \\ 0 & 4.625 & 4.75 & 1.625 \\ 4 & 9 & 2 & 3 \end{bmatrix}$

$C = \text{replace}(C, 3, -4, 1)$ $\qquad \begin{bmatrix} 1 & 1/8 & 3/4 & 1/8 \\ 0 & 4.625 & 4.75 & 1.625 \\ 0 & 8.5 & -1 & 2.5 \end{bmatrix}$

$C = \text{swap}(C, 2, 3)$ $\qquad \begin{bmatrix} 1 & 1/8 & 3/4 & 1/8 \\ 0 & 8.5 & -1 & 2.5 \\ 0 & 4.625 & 4.75 & 1.625 \end{bmatrix}$

$C = \text{scale}(C, 2, 1/8.5)$ $\qquad \begin{bmatrix} 1 & 1/8 & 3/4 & 1/8 \\ 0 & 1 & -.1176 & .2941 \\ 0 & 4.625 & 4.75 & 1.625 \end{bmatrix}$

$C = \text{replace}(C, 1, -1/8, 2)$ $\qquad \begin{bmatrix} 1 & 0 & .7647 & .0882 \\ 0 & 1 & -.1176 & .2941 \\ 0 & 4.625 & 4.75 & 1.625 \end{bmatrix}$

$C = \text{replace}(C, 3, -4.625, 2)$ $\qquad \begin{bmatrix} 1 & 0 & .7647 & .0882 \\ 0 & 1 & -.1176 & .2941 \\ 0 & 0 & 5.2941 & .2647 \end{bmatrix}$

$C = \text{scale}(C, 3, 1/5.2941)$ $\qquad \begin{bmatrix} 1 & 0 & .7647 & .0882 \\ 0 & 1 & -.1176 & .2941 \\ 0 & 0 & 1 & .05 \end{bmatrix}$

$C = \text{replace}(C, 2, .1176, 3)$ $\qquad \begin{bmatrix} 1 & 0 & .7647 & .0882 \\ 0 & 1 & 0 & .3 \\ 0 & 0 & 1 & .05 \end{bmatrix}$

$C = \text{replace}(C, 1, -.7647, 3)$ $\qquad \begin{bmatrix} 1 & 0 & 0 & .05 \\ 0 & 1 & 0 & .3 \\ 0 & 0 & 1 & .05 \end{bmatrix}$

So $\bar{x} = \begin{bmatrix} .05 \\ .3 \\ .05 \end{bmatrix} = \begin{bmatrix} 1/20 \\ 3/10 \\ 1/20 \end{bmatrix}$