Activity 2

1. (True or False) For any linear transformation $T: \mathbb{R}^n \to \mathbb{R}^k$, there exists a matrix A of appropriate size such that $T(\boldsymbol{x}) = A\boldsymbol{x}$ for any $\boldsymbol{x}$ in $\mathbb{R}^n$.

2. Let K be the set of vectors of the form $\begin{bmatrix} a+3c \\ 2b \\ 2a-b+c \end{bmatrix}$ with a, b, & c in $\mathbb{R}$. Then K is a

   a. 0-dimensional subspace of $\mathbb{R}^3$

   b. 1-dimensional subspace of $\mathbb{R}^3$

   c. 2-dimensional subspace of $\mathbb{R}^3$

   d. 3-dimensional subspace of $\mathbb{R}^3$

3. State the Rank Theorem for a matrix A.

4. Given that $\mathcal{B}$ is a set of 5 vectors in $\mathbb{R}^5$ that are linearly independent, which of the following are true statements:

   I. $\mathcal{B}$ contains 5 vectors.

   II. There is at least 1 vector in $\mathcal{B}$ that is a linear combination of the other vectors.

   III. Every vector in $\mathbb{R}^5$ can be written as a linear combination of the vectors of $\mathcal{B}$.

   IV. $\mathcal{B}$ is a basis of $\mathbb{R}^5$.

   V. $\mathcal{B}$ is the only basis of $\mathbb{R}^5$.

   a. I and II

   b. I, II, and III

   c. I, III, and IV

   d. I, III, IV, and V

5. Match the following matrices to the name that best describes them.

Upper triangular	$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$
Diagonal	$\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$
Block	$\begin{pmatrix} 2 & 3 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$
Echelon Form	$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$
Reduced Row Echelon Form	$\begin{pmatrix} 4 & 0 \\ 0 & -7 \end{pmatrix}$