

Key

ACTIVITY 2

True

1. (True or False) For any linear transformation $T: \mathbb{R}^n \rightarrow \mathbb{R}^k$, there exists a matrix A of appropriate size such that $T(x) = Ax$ for any x in \mathbb{R}^n .

2. Let K be the set of vectors of the form $\begin{bmatrix} a + 3c \\ 2b \\ 2a - b + c \end{bmatrix}$ with $a, b, & c$ in \mathbb{R} . Then K is a

$$\rightarrow = \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix} a + \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix} b + \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix} c$$
$$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 2 & 0 \\ 2 & -1 & 1 \end{bmatrix} \sim I_3$$

- a. 0-dimensional subspace of \mathbb{R}^3
- b. 1-dimensional subspace of \mathbb{R}^3
- c. 2-dimensional subspace of \mathbb{R}^3
- d. 3-dimensional subspace of \mathbb{R}^3

3. State the Rank Theorem for a matrix A .

$$\text{rank } A + \text{Nulity } A = \# \text{ of columns of } A$$

4. Given that \mathcal{B} is a set of 5 vectors in \mathbb{R}^5 that are linearly independent, which of the following are true statements:

- I. \mathcal{B} contains 5 vectors. ✓
- II. There is at least 1 vector in \mathcal{B} that is a linear combination of the other vectors. ✗
- III. Every vector in \mathbb{R}^5 can be written as a linear combination of the vectors of \mathcal{B} . ✓
- IV. \mathcal{B} is a basis of \mathbb{R}^5 . ✓
- V. \mathcal{B} is the only basis of \mathbb{R}^5 . ✗

- a. I and II
- b. I, II, and III
- c. I, III, and IV
- d. I, III, IV, and V

5. Match the following matrices to the name that best describes them.

however there are multiple answers

(a) Upper triangular

$$\begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$$

← only c

(b) Diagonal

$$\begin{pmatrix} 1 & 4 \\ 0 & -2 \end{pmatrix}$$

← a, c, & d

(c) Block

$$\begin{pmatrix} 2 & 3 & 0 \\ 0 & 0 & -1 \\ 0 & 0 & 0 \end{pmatrix}$$

← a, c, & d

(d) Echelon Form

$$\begin{pmatrix} 1 & 0 & -2 \\ 0 & 1 & 4 \\ 0 & 0 & 0 \end{pmatrix}$$

← a, c, d, & e

(e) Reduced Row Echelon Form

$$\begin{pmatrix} 4 & 0 \\ 0 & -7 \end{pmatrix}$$

← a, b, c, & d