Practice Problems
for Final

chapters :	1.1-1.5, 1.7-1.9	(see test 1)
chapters :	2.1 - 2.5, 3.1-3.2	(see midterm)
chapters :	3.3, 4.1 - 4.5	(see test 3)
chapters :	4.6 -4.7, 5.1- 5.4	(see below)

1. $$A = \begin{bmatrix} 1 & 1 & -3 & 7 & 9 & -9 \\ 1 & 2 & -4 & 10 & 13 & -12 \\ 1 & -1 & -1 & 1 & 1 & -3 \\ 1 & -3 & 1 & -5 & -7 & 3 \\ 1 & -2 & 0 & 0 & -5 & -4 \end{bmatrix}$$

a) Find rank A and Nulity A

b) Find dim row A and dim Col A^T

c) What is Col A? Row A? Nul A?

d) find a basis for each: Col A, row A, Nul A

2. a) If the null space of a 7×6 matrix A is 5-dimensional, what is the dimension of the column space of A?

b) If A is a 6×8 matrix what is the smallest posible dimension of Nul A?

c) If A is a 5×6 matrix with 4 pivot columns, what is rank A? What is Nulity A? Is Col $A = \mathbb{R}^4$? Why or Why not?

3. Let $\mathcal{B} = \{b_1, b_2, b_3\}$ and $\mathcal{C} = \{c_1, c_2, c_3\}$ be bases for a vector space V, and we know that

$c_1 = 2b_1 - b_2 + b_3$, $c_2 = 3b_2 + b_3$, $c_3 = -3b_1 + 2b_3$

a) Find the change of coordinates matrix from $\mathcal{C}$ to $\mathcal{B}$

b) Find $[x]_{\mathcal{B}}$ for $x = c_1 - 2c_2 + 2c_3$

4. $\mathcal{B}$ and $\mathcal{C}$ are bases of $\mathbb{R}^2$. Find the change of coordinates matrix from $\mathcal{B}$ to $\mathcal{C}$ and from $\mathcal{C}$ to $\mathcal{B}$.

$\mathcal{B} = \{b_1, b_2\}$ and $\mathcal{C} = \{c_1, c_2\}$ where

$$b_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}, \quad c_1 = \begin{bmatrix} 4 \\ 1 \end{bmatrix}, \quad c_2 = \begin{bmatrix} 7 \\ -2 \end{bmatrix}$$

5. $$A = \begin{bmatrix} 7 & 3 \\ 3 & -1 \end{bmatrix}$$

a) find the characteristic equation of A

b) give the eigenvalues of A

c) find the eigenspace associated to each λ

d) ~~give~~ what is the dimension of each eigenspace?

6. $A = \begin{bmatrix} -7 & -16 & 4 \\ 6 & 13 & -2 \\ 12 & 16 & 1 \end{bmatrix}$

a) find the characteristic equation of A

b) what are the eigenvalues of A?

c) find the eigenspace(s) associated to the eigenvalues of A.

d) What are the dimension(s) of the eigenspace(s)?

e) Is A diagonalizable? Why?

f) If A is diagonalizable, give it's diagonal factorization. If A is not diagonalizable, give/list a eigenvector ~~coor~~ coresponding to each eigenvalue.

7. $A = \begin{bmatrix} 4 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 2 \end{bmatrix}$

If A is diagonalizable, find P and D such that $A = PDP^{-1}$.
If not diagonalizable, explain why.

8. $A = PDP^{-1}$ where

$$A = \begin{bmatrix} 4 & -3 \\ 2 & -1 \end{bmatrix}, \quad P = \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}, \text{ and } \quad D = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$$

(a) compute A^{10} (Note: answer needs to be a single matrix)

(b) Let $T: \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation where $T(\bar{x}) = A\bar{x}$. Is there a basis $\mathcal{B}$ such that $[T]_{\mathcal{B}}$ is diagonal? If so, what is $\mathcal{B}$ and what is $[T]_{\mathcal{B}}$?

9. Compute A^5 where $A = \begin{bmatrix} 5 & 1 \\ 3 & 3 \end{bmatrix}$

10. Let $\mathcal{B} = \{b_1, b_2\}$ and $C = \{c_1, c_2\}$ be bases for the vector spaces V and W. Let $T: V \to W$ be a Linear transformation where

$$T(b_1) = 2c_1 - 4c_2 \quad \& \quad T(b_2) = 3c_1 + 7c_2$$

find the matrix for T relative to $\mathcal{B}$ and C.

11. find the $\mathcal{B}$-matrix for the transformation $T: \mathbb{R}^2 \to \mathbb{R}^2$ where $T(x) = Ax$ when

$$A = \begin{bmatrix} -1 & 4 \\ -2 & 3 \end{bmatrix}, \quad b_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -1 \\ 1 \end{bmatrix}, \quad \mathcal{B} = \{b_1, b_2\}$$