

# Quiz 10

For the matrix below, find its LU factorization by following the indicated steps.

$$A = \begin{bmatrix} 2 & 4 & 5 & -2 \\ -2 & -1 & -7 & 3 \\ 6 & 12 & 10 & -4 \\ -4 & -5 & 3 & -8 \end{bmatrix}$$

1. (7 points) Compute the matrices L and U

computing U

$$\begin{bmatrix} \textcircled{2} & 4 & 5 & -2 \\ -2 & -1 & -7 & 3 \\ 6 & 12 & 10 & -4 \\ -4 & -5 & 3 & -8 \end{bmatrix} \begin{array}{l} R_2 + R_1 \\ R_3 - 3R_1 \\ R_4 + 2R_1 \end{array}$$

$$\begin{bmatrix} 2 & 4 & 5 & -2 \\ 0 & \textcircled{3} & -2 & 1 \\ 0 & 0 & -5 & 2 \\ 0 & 3 & 13 & -12 \end{bmatrix} R_4 - R_2$$

$$\begin{bmatrix} 2 & 4 & 5 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & \textcircled{-5} & 2 \\ 0 & 0 & 15 & -13 \end{bmatrix} R_4 + 3R_3$$

$$\begin{bmatrix} 2 & 4 & 5 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -5 & 2 \\ 0 & 0 & 0 & -7 \end{bmatrix} = U$$

making L

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ -2 & 1 & -3 & 1 \end{bmatrix}$$

2. (3 points) Verify that the matrices L and U you found in part 1 satisfy  $A = L \cdot U$

$$L \cdot U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 3 & 0 & 1 & 0 \\ -2 & 1 & -3 & 1 \end{bmatrix} \begin{bmatrix} 2 & 4 & 5 & -2 \\ 0 & 3 & -2 & 1 \\ 0 & 0 & -5 & 2 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 5 & -2 \\ -2 & -1 & -7 & 3 \\ 6 & 12 & 10 & -4 \\ -4 & -5 & 3 & -8 \end{bmatrix}$$