

key

## Quiz 14

1. (2 points) Is  $\mathbb{R}^{217}$  a vector space?

yes

2. (4 points) Is  $\left\{ \begin{bmatrix} 2a-3c \\ a+b-c \\ 4c \end{bmatrix} : a, b, c \in \mathbb{R} \right\}$  a subspace of  $\mathbb{R}^3$ ? Show why or why not.

yes, b/c

$$\left\{ \begin{bmatrix} 2a-3c \\ a+b-c \\ 4c \end{bmatrix} : a, b, c \in \mathbb{R} \right\} = \text{span} \left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ -1 \\ 4 \end{bmatrix} \right\}$$

$$\text{Since } \begin{bmatrix} 2a-3c \\ a+b-c \\ 4c \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} a + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} b + \begin{bmatrix} -3 \\ -1 \\ 4 \end{bmatrix} c$$

and we know  $\text{span}\{ \}$  is a subspace

3. (4 points) Describe the Null Space of  $A = \begin{bmatrix} 1 & 0 & 5 \\ 2 & -3 & 1 \end{bmatrix}$ ?

$$A\bar{x} = \bar{0}$$

$$\leadsto \left[ \begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 2 & -3 & 1 & 0 \end{array} \right] \xrightarrow{\text{ref}} \left[ \begin{array}{ccc|c} 1 & 0 & 5 & 0 \\ 0 & 1 & 3 & 0 \end{array} \right]$$

$$\Rightarrow \begin{cases} x_1 + 5x_3 = 0 \\ x_2 + 3x_3 = 0 \end{cases}$$

and we know  $\bar{x} \in \mathbb{R}^3$

$$\Rightarrow \bar{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5x_3 \\ -3x_3 \\ x_3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix} x_3$$

which means

$$\text{Nul}(A) = \text{span} \left\{ \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix} \right\} = \left\{ \begin{bmatrix} -5 \\ -3 \\ 1 \end{bmatrix} a \mid a \in \mathbb{R} \right\}$$