Key

QUIZ 17

1. Let $\mathcal{B} = \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} \right\}$ be a basis for $\mathbb{R}^3$. Suppose that $[x]_{\mathcal{B}} = \begin{bmatrix} 3 \\ -2 \\ 11 \end{bmatrix}$ and $y = \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix}$

   a. (3 points) Find the vector x in the standard basis

$$X = 3\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} - 2\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + 11\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 24 \\ 14 \end{bmatrix}$$

   b. (3 points) Compute $[y]_{\mathcal{B}}$

$$c_1\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + c_2\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix} + c_3\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 4 \\ -2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 1 & 0 & -3 \\ 0 & -1 & 2 & 4 \\ 1 & 0 & 1 & -2 \end{bmatrix} \xrightarrow{rref} \begin{bmatrix} 1 & 0 & 0 & -5 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$$

$$[y]_{\mathcal{B}} = \begin{bmatrix} -5 \\ 2 \\ 3 \end{bmatrix}$$

2. (4 points) Determine whether the polynomials $(2 + t^2)$, $(-t + 3t^3)$, $(1 - t)$, and $(t^2 + t^3)$ are linearly independent in $\mathbb{P}_3$ or not.

$$c_1(2+t^2) + c_2(-t+3t^3) + c_3(1-t) + c_4(t^2+t^3) = 0$$

$$(2c_1 + c_3) + (-c_2 - c_3)t + (c_1 + c_4)t^2 + (3c_2 + c_4)t^3 = 0$$

$$\Rightarrow \begin{cases} 2c_1 + c_3 = 0 \\ -c_2 - c_3 = 0 \\ c_1 + c_4 = 0 \\ 3c_2 + c_4 = 0 \end{cases}$$

$$\begin{bmatrix} 2 & 0 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 0 & 3 & 0 & 1 & 0 \end{bmatrix} \xrightarrow{rref} \left[I_4 \mid \bar{0} \right]$$

Since $c_1 = c_2 = c_3 = c_4 = 0$ is the only solution these 4 polynomials are linearly independent in $\mathbb{P}_3$